Outcome missingness in principal stratification: in defense of MAR over latent ignorability (latent MAR)

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Context

- The study of treatment effects is often complicated by noncompliance
- Principal stratification (Frangakis & Rubin, 2002): Define strata of people based on potential values of treatment received
- Our focus: outcome missingness within this framework
  - Revisit a standard assumption: latent ignorability (latent MAR)
  - Conclude that MAR should be preferred

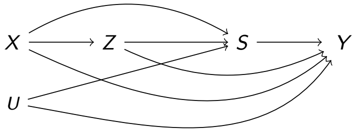
Principal stratification on one slide – putting aside missingness

Z treatment assigned (binary)
S treatment received (binary)
Y outcome

X baseline covariates
U unobserved confounders of S, Y

Y_z potential outcomes
S_z potential treatment received
C principal stratum, $C := (S_1, S_0)$

PCEs: $\mathrm{E}[Y_1 - Y_0 \mid C = c]$



DAG for a simple main model

One-sided noncompliance:

$$C = \begin{cases} \text{complier} & \text{if } S_1 = 1 \\ \text{noncomplier} & \text{if } S_1 = 0 \end{cases}$$

Two-sided noncompliance:

$$C = \begin{cases} \text{complier} & \text{if } S_1 = 1, S_0 = 0 \\ \text{defier} & \text{if } S_1 = 0, S_0 = 1 \\ \text{always-taker} & \text{if } S_1 = S_0 = 1 \\ \text{never-taker} & \text{if } S_1 = S_0 = 0 \end{cases}$$

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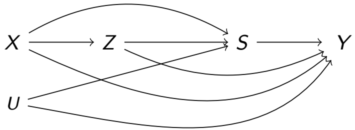
Identifying assumptions

- consistency/SUTVA
- ignorability (and positivity) of treatment assigned Z
- monotonicity, $S_1 \geq S_0$ (ie no defiers)
- approach-specific assumption (eg ER, PI)



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A bit more on identification

Consider one-sided noncompliance.

Standard causal inference assumptions (consistency and treatment assignment ignorability and positivity) provide

$$\mathrm{CACE} = \mathrm{E}_{X \mid \mathrm{complier}}\{\mathrm{E}[Y \mid X, Z = 1, \mathrm{complier}] - \mathrm{E}[Y \mid X, Z = 0, \mathrm{complier}]\},$$
$$\mathrm{NACE} = \mathrm{E}_{X \mid \mathrm{noncomplier}}\{\mathrm{E}[Y \mid X, Z = 1, \mathrm{noncomplier}] - \mathrm{E}[Y \mid X, Z = 0, \mathrm{noncomplier}]\}.$$

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The approach-specific assumption (ER or PI) helps disentangle the two pieces from the mixture.

Outcome missingness complicates all this

R: variable indicating Y is observed ($R = 1$) or missing ($R = 0$)

To recover identification requires some assumption about the missingness

but which assumption?

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but which assumption?

MAR: $R \perp\!\!\!\perp Y \mid X, Z, S$

Latent ignorability (latent MAR): $R \perp\!\!\!\perp Y \mid X, Z, C$

– seen as a relaxation of MAR

History of LI/LMAR ($R \perp\!\!\!\perp Y \mid X, Z, C$)

- First used in Baker 1998 and formalized in Frangakis and Rubin 1999
- Picked up by many authors, eg Peng, Little, and Raghunathan 2004; Mealli et al. 2004; Dunn, Maracy, and Tomenson 2005; Zhou and Li 2006; Taylor and Zhou 2009; Chen, Geng, and Zhou 2009; Jo, Ginexi, and Ialongo 2010; Lui and Chang 2010; Mealli and Mattei 2012; Chen et al. 2015; Nguyen, Carlson, and Stuart 2024
- Most have used LI/LMAR within the instrumental variable approach, combining it with an additional assumption
  - most often an ER on response, labeling the combination of the two ERs *compound ER*
  - Mealli et al. 2004; Jo, Ginexi, and Ialongo 2010 propose alternative of stable complier response
- We (Nguyen, Carlson, and Stuart 2024) generalize to allow non-IV approaches (eg PI) and a wider range of specific missingness assumptions
- BUT the plausibility and necessity of LI/LMAR has not been examined

What we find

LI/LMAR is not a relaxation of MAR. Where LI/LMAR holds, MAR also holds.

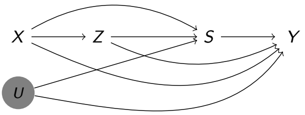
For the sake of breaking dependence between Y and R, no benefit is gained from conditioning on C on top of observed variables.

We then turn to focus on MAR, clarifying conditions on the causal structure for MAR to hold, and recover identification within the IV and PI approaches.

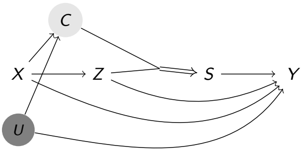
Bringing principal stratum into the causal graph

Change representation of model for S

- (Z, C) perfectly determine S
- The influence on S of all causes other than Z go through C

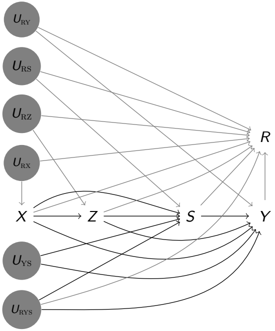


D: DAG for the simple main model

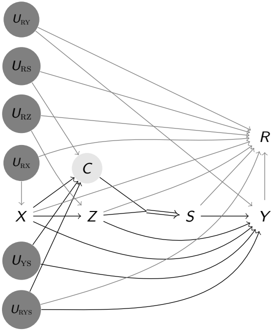


G. Principal stratification graph based on D

Introducing outcome missingness (and no assumption yet!)



DM: D + semi-saturated missingness model



GM: Principal stratification graph based on DM

Conditional graphs

3 rules

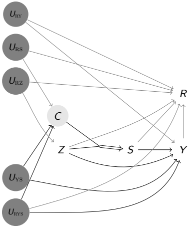
1. follow the causal order
2. if the variable conditioned on is a collider of 2+ causes, indicate the induced non-causal dependence between the causes (using undirected dashed edges)
3. drop the variable being conditioned on and all the arrows and edges involving it

then some decluttering

- drop any remaining variable that is a constant (with its arrows and edges)
- combine in one node any pair of adjacent variables with a one-to-one correspondence
- drop any unobserved variable that has become a unique cause of a single variable and is otherwise not connected to the rest of the graph
- drop any unobserved variable that is not a cause of any other variables

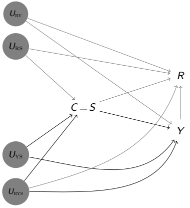
Conditional graphs based on GM

conditional on X

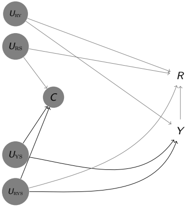


GMx:
Conditional graph given $X = x$

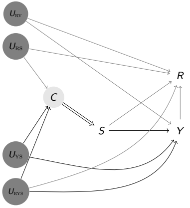
conditional on X, Z



GMx1:
Conditional graph given $(X = x, Z = 1)$,
one-sided noncompliance setting



GMx0:
Conditional graph given $(X = x, Z = 0)$,
one-sided noncompliance setting



GMxz:
Conditional graph given $(X = x, Z = z)$,
two-sided noncompliance setting

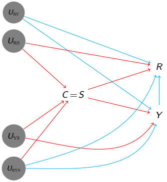
Types of paths via which R and Y are dependent

One-sided noncompliance setting: LMAR = MAR1 + LMAR0

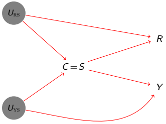
Types of paths via which R and Y are dependent

One-sided noncompliance setting: LMAR = MAR1 + LMAR0

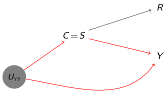
Treatment arm



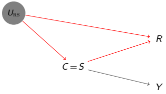
GMx1
(same as GMx1 in Fig. **??**C):
MNAR1



GMx1-a
(no direct path, no U_{RY}/U_{RYS}):
MNAR1



GMx1-b
(no direct path, no U_{RY}/U_{RYS}, no U_{RS}):
MAR1

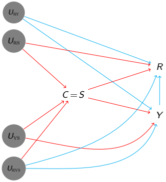


GMx1-c
(no direct path, no U_{RY}/U_{RYS}, no U_{YS}):
MAR1

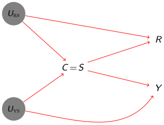
Types of paths via which R and Y are dependent

One-sided noncompliance setting: LMAR = MAR1 + LMAR0

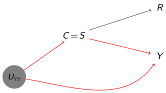
Treatment arm



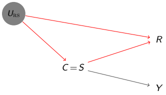
GMx1
(same as GMx1 in Fig. **??**C):
MNAR1



GMx1-a
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$):
MNAR1

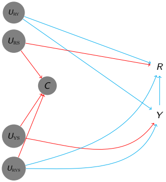


GMx1-b
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$, no $U_{\rm RS}$):
MAR1

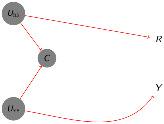


GMx1-c
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$, no $U_{\rm YS}$):
MAR1

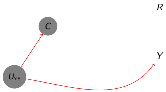
Control arm



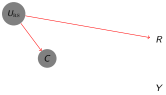
GMx0
(same as GMx0 in Fig. **??**C):
MNAR0



GMx0-a
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$):
MAR0, not LMAR0



GMx0-b
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$, no $U_{\rm RS}$):
LMAR0, MAR0



GMx0-c
(no direct path, no $U_{\rm RY}/U_{\rm RYS}$, no $U_{\rm YS}$):
LMAR0, MAR0

Main finding 1

LI?LMAR is not a relaxation of MAR.

For the purpose of rendering R and Y conditionally independent (or reduce their dependence), it is not necessary to condition on C on top of X, Z, S.
Conditioning on C may induce unwanted dependence.

C should not be conditioned on in missingness assumptions.

We should let go of LI/LMAR (and all the specific assumptions that have accompanied LMAR), and instead embrace MAR.

If MAR is deemed unlikely, should use standard strategies to handle MNAR instead of adopting LMAR.

Outline

A deep dive into MAR

Then a deep dive into MAR $\rightarrow$ Main finding 2

Conditions for MAR to hold – without auxiliary variables

- no direct path
- no triangle (unobserved common causes of Y and R)
- no S-butterfly
- no X-butterfly

Then a deep dive into MAR → Main finding 2

Conditions for MAR to hold – without auxiliary variables

- no direct path
- no triangle (unobserved common causes of Y and R)
- no S-butterfly
- no X-butterfly

Conditions for MAR to hold – with auxiliary variables W

- no/controlled direct path
- no/controlled triangle
- no/controlled S-butterfly
- no/controlled X-butterfly
- no W-butterfly

"controlling" is by conditioning on W

so where W variables are in the causal structure is important

Additional results

MAR-based recovery of effect identification

- within the IV approach
- within the PI approach

No need for a specific missingness assumption

Conclusion

We should put LI/LMAR aside and work with MAR (and standard MNAR).

More generally, should be careful with assumptions that condition on principal stratum.