Causal mediation analysis: from simple to more robust estimation strategies

Trang Quynh Nguyen

Johns Hopkins Bloomberg School of Public Health | trang.nguyen@jhu.edu

joint work with Elizabeth Stuart, Elizabeth Ogburn, Ian Schmid, Elizabeth Parker, Noah Greifer and Ina Koning

presentation @ CMStatistics 2021

session EO770: Causal Mediation Analysis, 2021-12-18

organizer & chair: Anita Lindmark

Motivation

- Mediation analysis is popular
- The methodological literature on causal mediation analysis is exciting, fast growing, and highly technical
  - Different types of effects
  - Assumptions
  - Lots of methods
- Recent method reviews (e.g., Vo et al., Stuart et al.) of applied mediation analyses (up until 2020)
  - Uptake slow, most still use traditional mediation analysis
  - Also lots of basic problems: temporality, confounding
- Concern about “easy” mediation analysis, especially with software that “does it for you”
  - True with traditional mediation analysis
  - Also seen in recent papers that use causal mediation analysis

Motivation

- Need to help ground practitioners in some basics

- We are not the only ones thinking this
  - e.g., the AGReMA statement (Hopin et al.) -- guideline on reporting of mediation analysis

- Our project (PI Stuart) aims to bring causal mediation analysis to mental health researchers

A series of 3 papers

- Estimands: define effects based on what we want to learn (Psych Methods)
- Identification: handle the range of effects via five potential outcome types
- Estimation: build intuitive appreciation for options including simpler and more robust methods (this talk)

Key ideas of paper

- Use two ingredients that are familiar
  - weighting
  - regression
- Treat the identification result/estimation task as a puzzle
  - find solutions using the tools
  - visualization helpful
- Solutions may be simple or complex
  - simpler – nonrobust
  - more complex (combining tools) – more robust
- User-friendliness
  - appeal to intuition
  - based on theory, but can hide theory

Scope of paper

- Simple setting

  A binary exposure

  M mediator

  Y univariate outcome

  C pre-exposure covariates

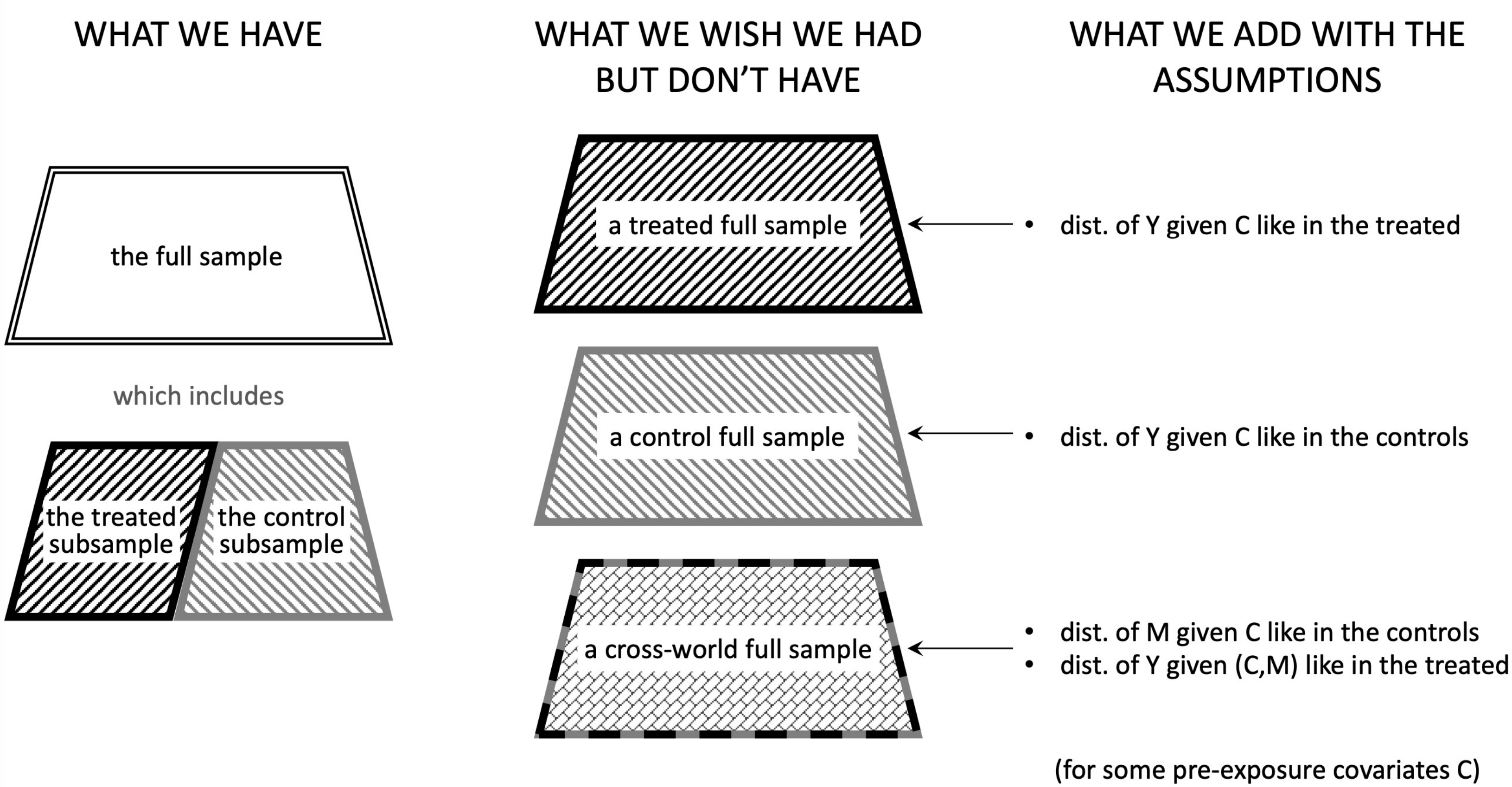
- Estimand: marginal natural (in)direct effects
  - contrasting the means of Y_1, Y_0 and Y_{1M_0} (or Y_{0M_1})
- Assume effects are identified
- Alternative if don't like natural effects or the cross-world assumption
  - consider hypothetical intervention on exposure and mediator distribution
  - in simple case with no intermediate confounder
- The ideas apply to other estimands
  - solve another puzzle!

ID result/estimation task as a puzzle

Under the assumptions, the relevant potential outcome means are identified as

$$E[Y_1] = E_C\{E[Y|C, A = 1]\}$$
$$E[Y_0] = E_C\{E[Y|C, A = 0]\}$$
$$E\left[Y_{1M_0}\right] = E_C(E_{M|C,A=0}\{E[Y|C, A = 1, M]\})$$

ID result/estimation task as a puzzle



Tool #1: weighting

- Form relevant pseudo samples
  - pseudo treated sample
  - pseudo control sample
  - pseudo cross-world sample
  - etc.
- Use:
  - weighting based estimation
    - requires that the weights be consistently estimated
    - check balance
  - can combine with tool #2

Pseudo treated/control samples

- Inverse probability weighting
  - For treated units: $\frac{1}{P(A=1|C)}$
  - For control units: $\frac{1}{P(A=0|C)}$

Pseudo cross-world sample

- Formed out of treated units
  - so Y given C,M dist. is that of the treated
- Also need
  - distribution of C like in full sample
  - distribution of M given C like in control units

Pseudo cross-world sample

3 equivalent expressions of the weight function

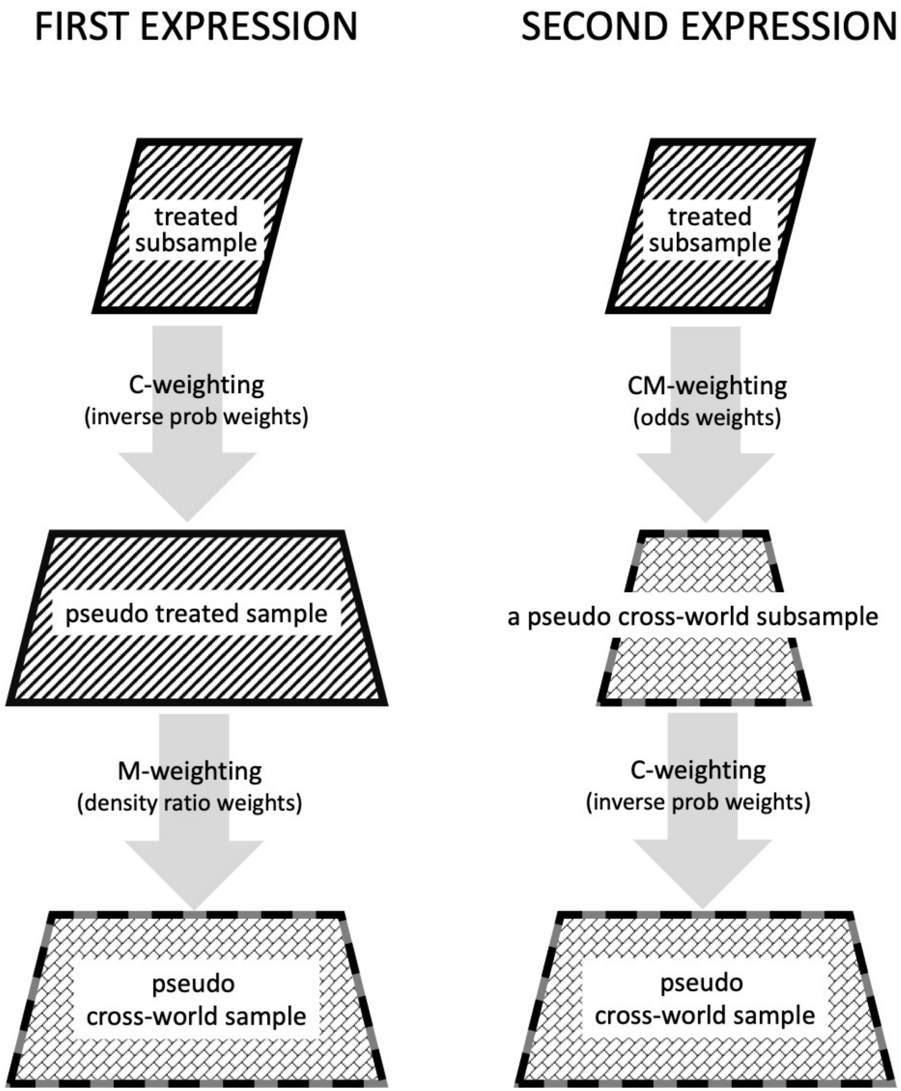
- First expression (Hong, 2010)

$$\frac{1}{P(A=1|C)}\frac{P(M|C,A=0)}{P(M|C,A=1)}$$

- Second expression due to a connection b/w the mediator density ratio with a ratio of two odds (Zheng & van der Laan 2012, Huber 2014)

$$\frac{P(A=0|C,M)}{P(A=1|C,M)}\frac{1}{P(A=0|C)}$$

Views from 3 expressions of the cross-world weights



Pseudo cross-world sample

3 equivalent expressions of the weight function

- First expression (Hong, 2010)

$$\frac{1}{P(A=1|C)}\frac{P(M|C,A=0)}{P(M|C,A=1)}$$

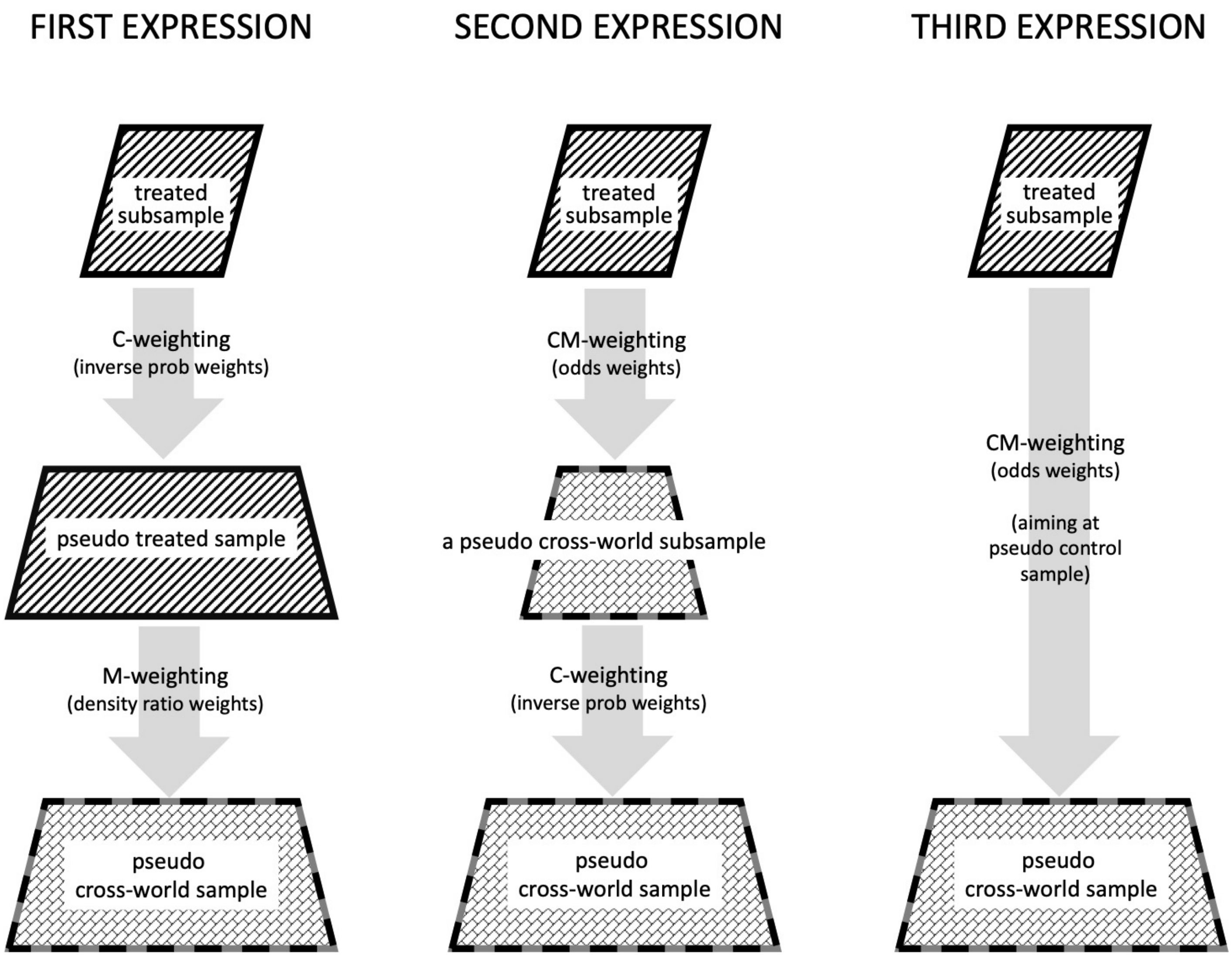
- Second expression due to a connection b/w the mediator density ratio with a ratio of two odds (Zheng 2012, Huber 2014)

$$\frac{P(A=0|C,M)}{P(A=1|C,M)}\frac{1}{P(A=0|C)}$$

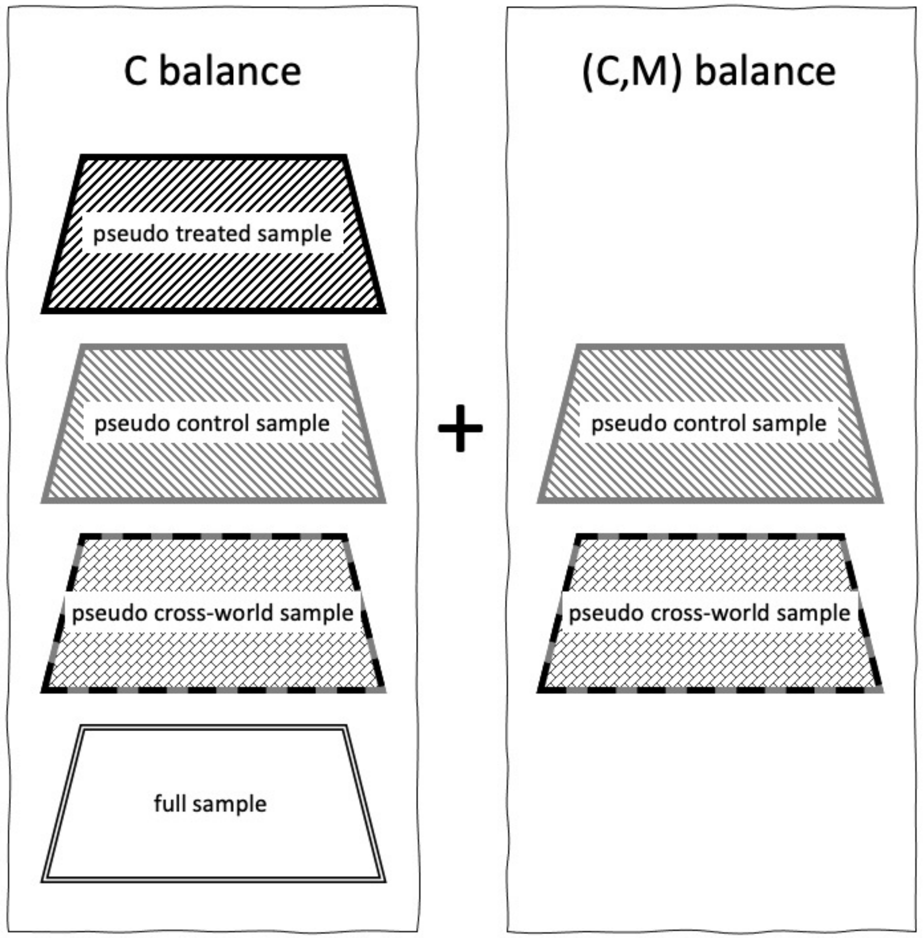
- We found a new third expression (shown in stabilized form)

$$\frac{P(C,M|A=0)\dfrac{P(A=0)}{P(A=0|C)}}{P(C,M|A=1)}$$

Views from 3 expressions of the cross-world weights



Desired balance



Tool #2: regression

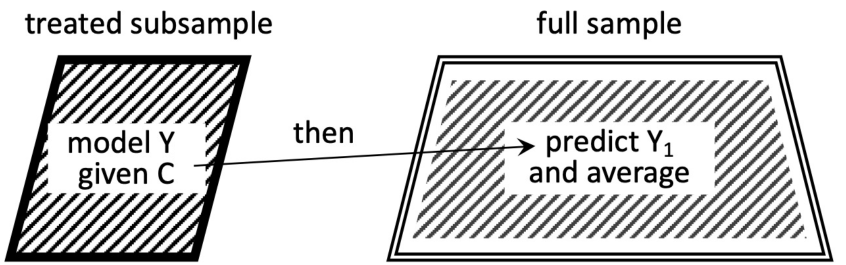
- specifically, regression-based prediction (or simulation)

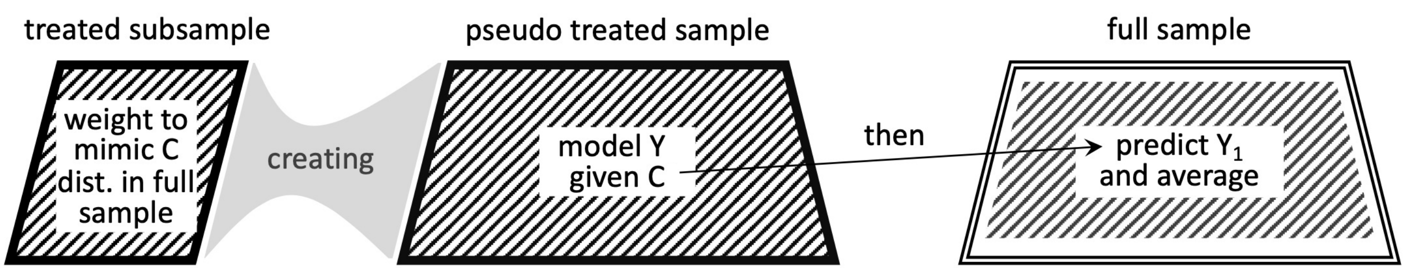
- can be used alone or combined with weighting
- some combinations induce robustness

Estimators in pairs

- A simpler estimator
  - solves the puzzle
  - requires all modeling components to be consistent
- A more complex version
  - replaces all subsamples used to fit models with relevant pseudo samples
    - fit model to predictors space where model is used for prediction
  - requires regression model (for prediction) to satisfy mean recovery
    - even if predictions are wrong, they will be right on average (if weights are correct)
  - more robust: ok if one of two components (weights or regression) correct

Estimating $E[Y_1]$





Estimating $E[Y_{1M_0}]$

multiple solutions, with different properties

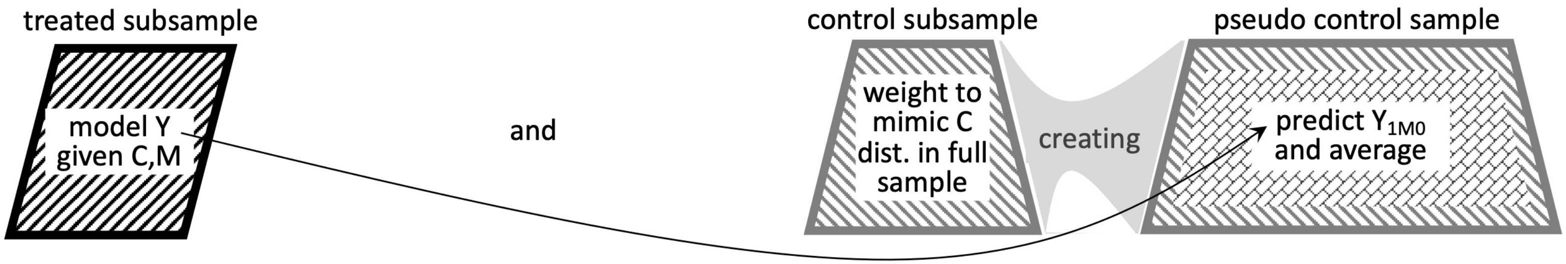
we'll show 4 pairs

with each pair, we'll note

- simpler estimator: estimating components it relies on
- more complex estimator: the specific robustness (and nonrobustness)

Estimating $E[Y_{1M_0}]$

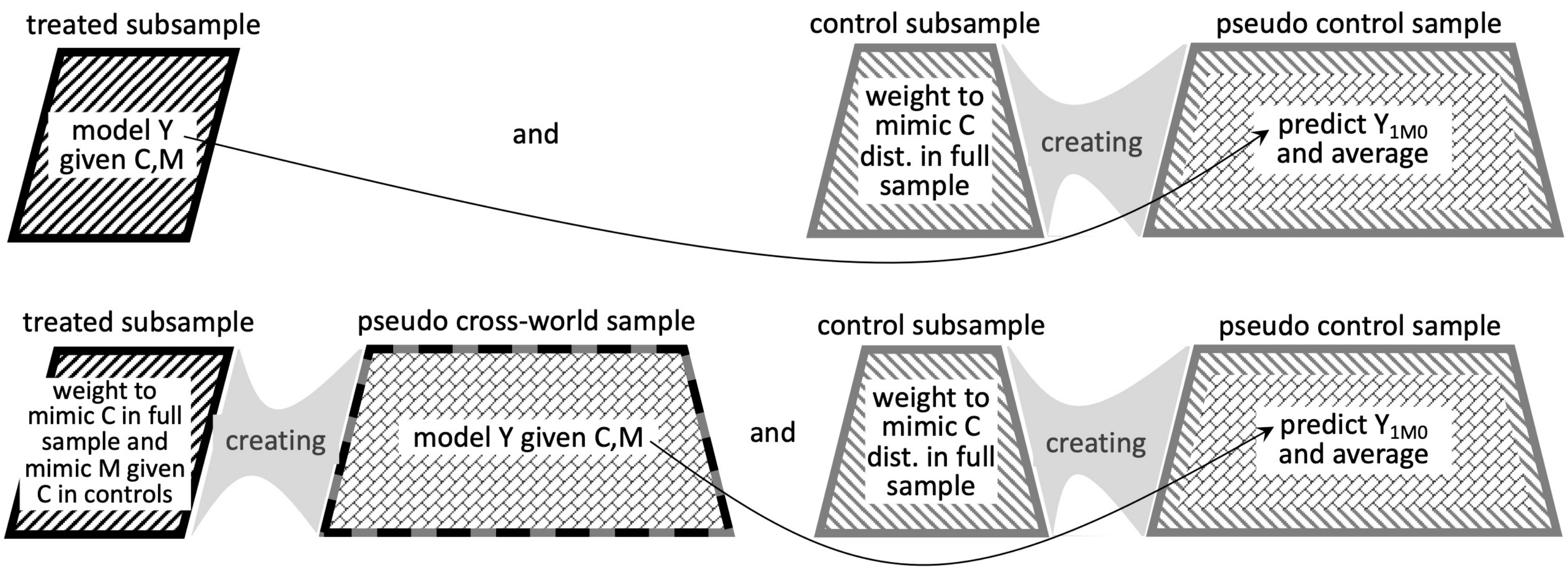
“outcome imputation” method



simpler version

- control weights
- outcome regression

Estimating $E[Y_{1M_0}]$



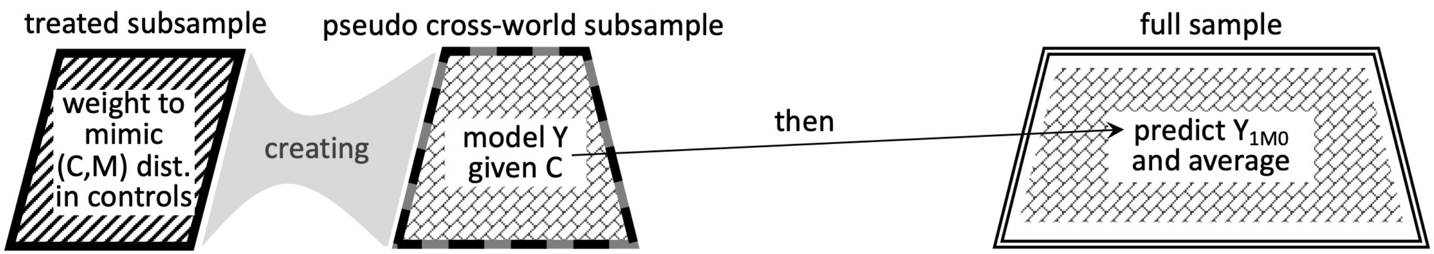
simpler version

- control weights
- outcome regression

more robust version

- control weights
- either outcome regression or cross-world weights

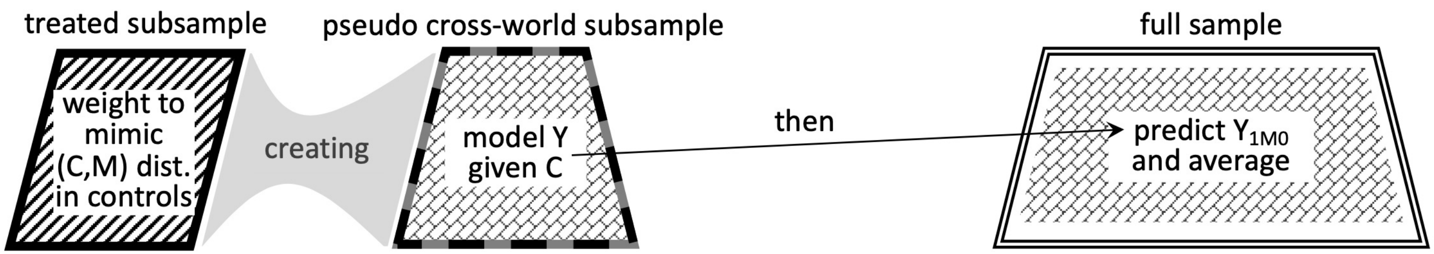
Estimating $E\left[Y_{1M_0}\right]$

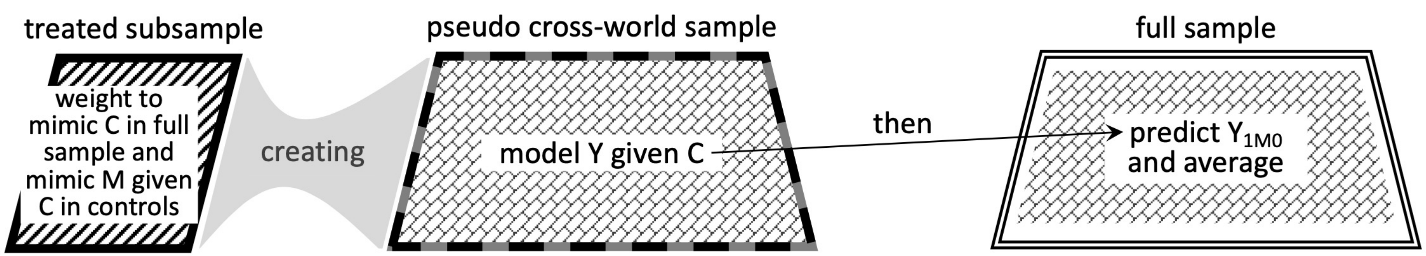


simpler version

- the weights
- outcome model

Estimating $E[Y_{1M_0}]$





simpler version

- the weights
- outcome model

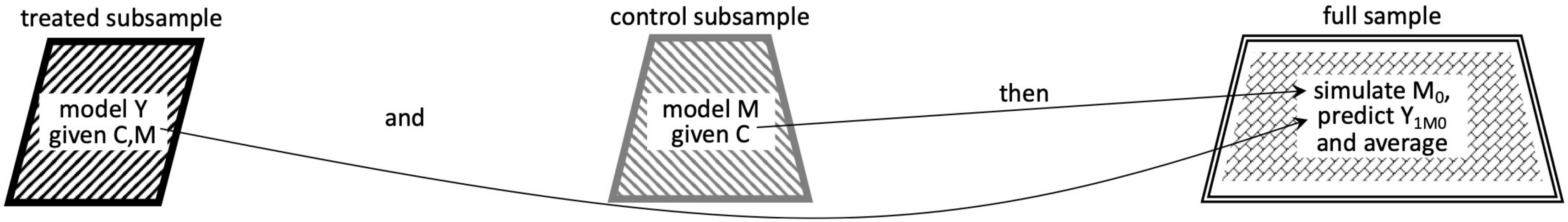
more robust version

either the weights only

or the M-part of the weights + outcome model

Estimating $E[Y_{1M_0}]$

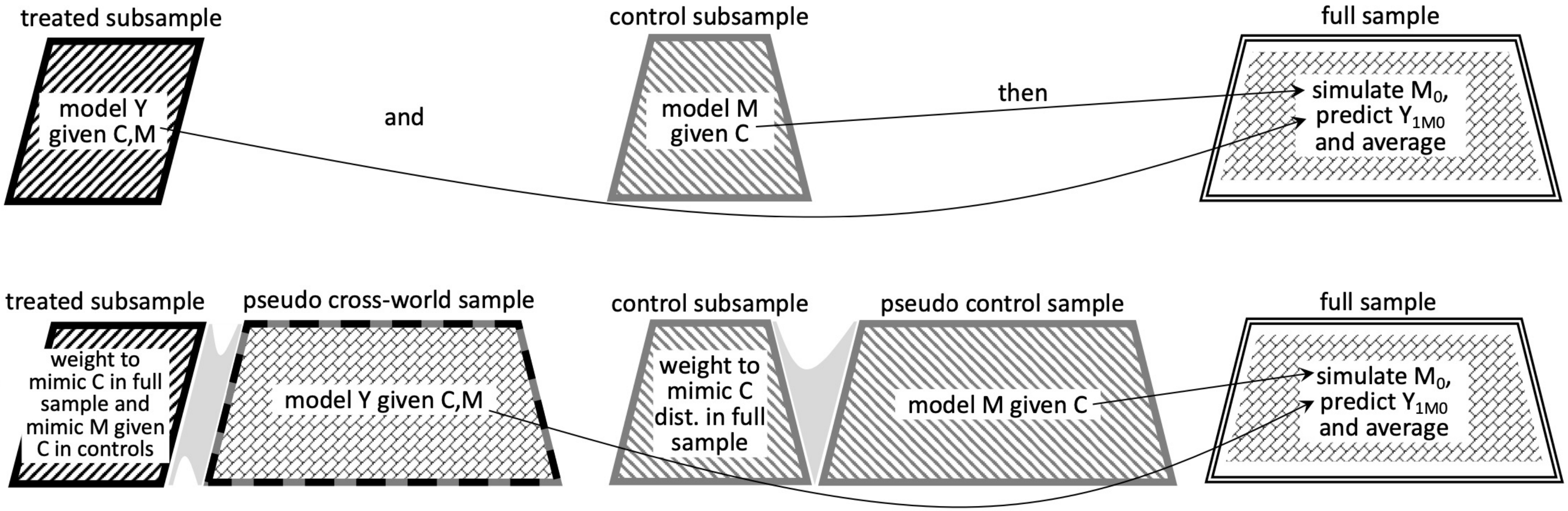
Imai's mediator simulation approach



simpler version

- mediator density
- outcome regression

Estimating $E[Y_{1M_0}]$



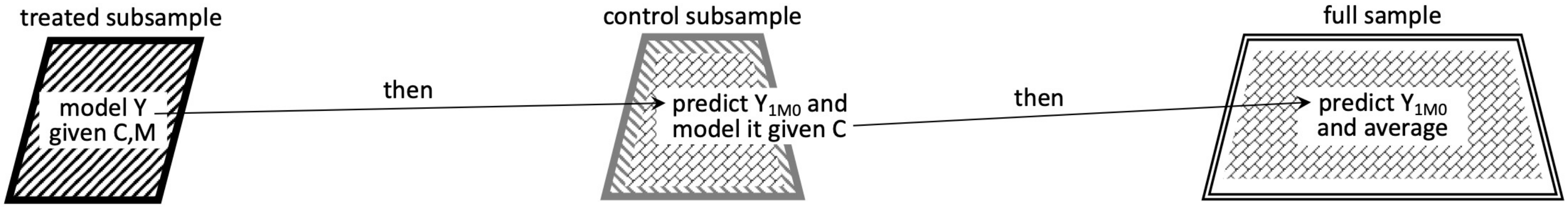
simpler version

- mediator density
- outcome regression

more robust version

- mediator density
- either outcome model or cross-world weights

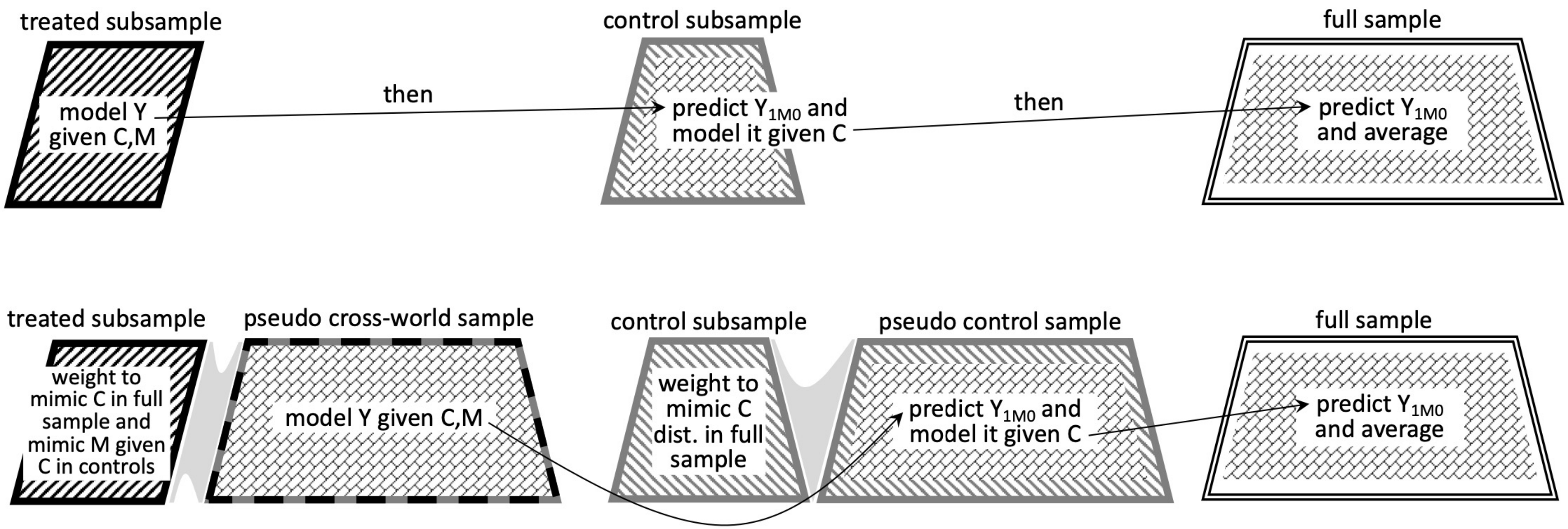
Estimating $E[Y_{1M_0}]$



simpler version

- two regression models

Estimating $E[Y_{1M_0}]$



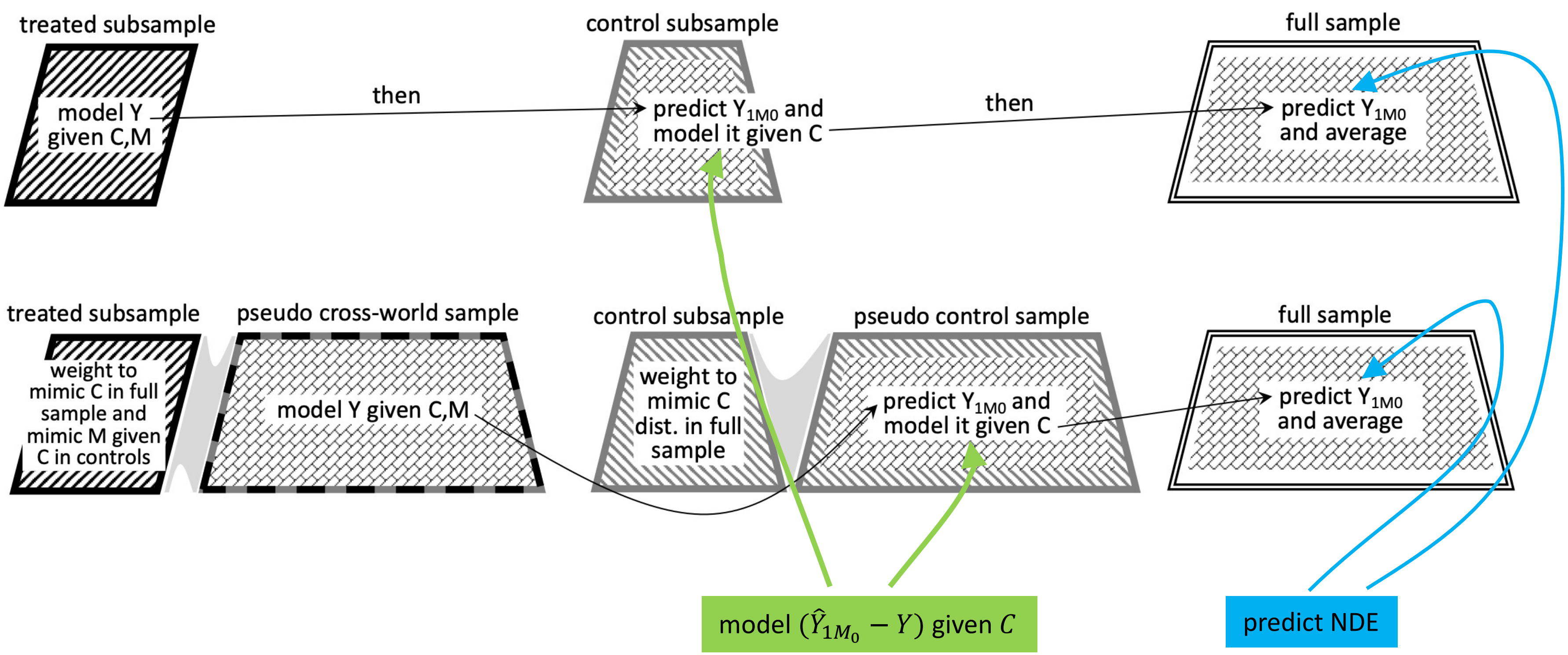
simpler version

- two regression models

robust version

- either first model or cross-world weights
- either second model or control weights

If target marginal *additive* effects, can modify last pair to estimate NDE $= E[Y_{1M_0} - Y_0]$



related to an estimator in Zheng & van der Laan 2012

- There are other options and additional strategies
- Not all, but a lot can be communicated and appreciated using this practitioner-centric lens

Some thoughts looking forward

- General
  - Causal mediation analysis will be done a lot more frequently, and more will be done by people who are not causal mediation methodologists
  - It’s super hard (I fail all the time)
    but very important to seek a language that more people understand

- Specific
  - We have done this for one puzzle. There remain puzzles for other effects, especially the diverse range of interventional effects flexibly defined