Sensitivity analysis for principal ignorability violation in estimating complier and noncomplier average causal effects

Trang Quynh Nguyen

joint work with Liz Stuart, Dan Scharfstein, Betsy Ogburn
(but all problems mine)

Johns Hopkins causal inference working group 2023-02-28

Outline

Problem: Noncompliance/post-treatment events

The study of treatment effects often complicated by noncompliance or other significant post-treatment events, eg

- JOBS II study[1]: nonattendance of training
- Head Start[2]: families in either arm may enroll kid or not
- Substance use treatment modalities[3]: institutionalization
- Treatment effect on quality of life[4]: death

[1] Amiram D. Vinokur, Richard H. Price, and Yaacov Schul. "Impact of the JOBS intervention on unemployed workers varying in risk for depression". In: *American Journal of Community Psychology* 23.1 (1995), pp. 39–74. DOI: 10.1007/BF02506922.

[2] Avi Feller, Fabrizia Mealli, and Luke Miratrix. "Principal Score Methods: Assumptions, Extensions, and Practical Considerations". In: *Journal of Educational and Behavioral Statistics* 42.6 (2017), pp. 726–758. DOI: 10.3102/1076998617719726.

[3] Beth Ann Griffin, Daniel F. McCaffrey, and Andrew R. Morral. "An application of principal stratification to control for institutionalization at follow-up in studies of substance abuse treatment programs". In: *The Annals of Applied Statistics* 2.3 (2008), pp. 1034–1055. DOI: 10.1214/08-AOAS179.

[4] Donald B. Rubin. "Causal inference through potential outcomes and principal stratification: Application to studies with "censoring" due to death". In: *Statistical Science* 21.3 (2006), pp. 299–309. DOI: 10.1214/088342306000000114.

Problem: Noncompliance/post-treatment events

The study of treatment effects often complicated by noncompliance or other significant post-treatment events

In some of these cases, the ATE may no longer be of interest (or defined)

Researchers might be interested in the effect of receiving treatment, or effect of treatment given a post-treatment event

but would break randomization, compare apples to oranges

Principal stratification

Define groups, termed *principal strata*, based on potential values of the post-treatment variable[1] (treatment received)

Notation

- Z treatment assigned (binary)
- Y outcome; Y_1, Y_0 potential outcomes
- S treatment received; S_1, S_0 potential treatment received
- X baseline covariates
- C principal stratum, defined based on (S_1, S_0), is a pre-treatment variable

Principal causal effects: $\mathrm{E}[Y_1 - Y_0 \mid C]$

[1] Constantine E. Frangakis and Donald B. Rubin. "Principal Stratification in Causal Inference". In: *Biometrics* 58.1 (2002), pp. 21–29. DOI: 10.1111/j.0006-341X.2002.00021.x.

The setting we focus on

There are different settings, depending on

- the type of variable S is: binary or other
- using S_1 only or both S_1, S_0 (one-sided or two-sided noncompliance)
- Y is well-defined generally or conditional on S

Our focus: binary S + one-sided noncompliance + Y well-defined generally

interested in complier and noncomplier average causal effects (CACE, NACE)

$$\Delta_c := \mathrm{E}[Y_1 - Y_0 \mid C = c] \text{ for } c = 1, 0$$

illustrative example: JOBS II

- unemployed workers are randomized to either a one-week training aiming to improve job searching skills and mental health (*intervention*) or to receive a booklet with job search tips (*control*)
- about half of those assigned to intervention didn't attend
- outcomes: work, earnings, depressive symptoms

Outline

The identification challenge: C is only partially observed

$$\Delta_c = \mathrm{E}[Y_1 \mid C = c] - \mathrm{E}[Y_0 \mid C = c]$$

$\mathrm{E}[Y_1 \mid C = c]$ is identified under the standard causal inference assumptions

A0 (consistency):	$Y = ZY_1 + (1 - Z)Y_0,\ S = ZC$
A1 (treatment assignment ignorability):	$Z \perp\!\!\!\perp (Y_1, Y_0, C) \mid X$
A2 (treatment assignment positivity):	$0 < \mathrm{P}(Z = 1 \mid X) < 1$

but $\mathrm{E}[Y_0 \mid C = c]$ is not identified under these assumptions alone

Several identification strategies

- Instrumental variable strategy[2]
  - rely on the *exclusion restriction* (ER) assumption, which implies $\tau_{00} = \tau_{10}$ and NACE $= 0$
  - not appropriate if treatment receipt not strictly binary[3]
    - compliance is defined using one important component of the intervention
    - dichotomizing a continuous participation variable
  - does not allow psychological effects, effects of compensating behaviors, etc.

[2] Joshua D. Angrist and Guido W. Imbens. "Two-stage least squares estimation of average causal effects in models with variable treatment intensity". In: *Journal of the American Statistical Association* 90.430 (1995), pp. 431–442. DOI: 10.1080/01621459.1995.10476535.

[3] John Marshall. "Coarsening Bias: How Coarse Treatment Measurement Upwardly Biases Instrumental Variable Estimates". en. In: *Political Analysis* 24.2 (2016), pp. 157–171. DOI: 10.1093/pan/mpw007. (Visited on 02/05/2023), Martin E Andresen and Martin Huber. "Instrument-based estimation with binarised treatments: issues and tests for the exclusion restriction". In: *The Econometrics Journal* 24.3 (Sept. 2021), pp. 536–558. DOI: 10.1093/ectj/utab002. (Visited on 02/05/2023).

Several identification strategies

- Instrumental variable strategy
  - rely on the *exclusion restriction* (ER) assumption, which implies $\tau_{00} = \tau_{10}$ and NACE $= 0$
  - not appropriate if treatment receipt not strictly binary
    - compliance is defined using one important component of the intervention
    - dichotomizing a continuous participation variable
  - does not allow psychological effects, effects of compensating behaviors, etc.

- Principal ignorability (PI) strategy[2]
  - PI assumption: $C \perp\!\!\!\perp Y_0 \mid X$, or $\mathrm{E}[Y_0 \mid X, C = 1] = \mathrm{E}[Y_0 \mid X, C = 0]$
  - appealing if have reach covariate data
  - justifies the *principal score weighting* method
  - does not assume NACE $= 0$, thus may be useful where ER is not justified

[2] Elizabeth A. Stuart and Booil Jo. "Assessing the sensitivity of methods for estimating principal causal effects". In: *Statistical Methods in Medical Research* 24.6 (2015), pp. 657–674. DOI: 10.1177/0962280211421840, Avi Feller, Fabrizia Mealli, and Luke Miratrix. "Principal Score Methods: Assumptions, Extensions, and Practical Considerations". In: *Journal of Educational and Behavioral Statistics* 42.6 (2017), pp. 726–758. DOI: 10.3102/1076998617719726, Peng Ding and Jiannan Lu. "Principal stratification analysis using principal scores". In: *Journal of the Royal Statistical Society. Series B: Statistical Methodology* 79.3 (2017), pp. 757–777. DOI: 10.1111/rssb.12191. arXiv: 1602.01196, Zhichao Jiang, Shu Yang, and Peng Ding. "Multiply robust estimation of causal effects under principal ignorability". en. In: *Journal of the Royal Statistical Society. Series B: Statistical Methodology* 84.4 (2022), pp. 1423–1445. DOI: 10.1111/rssb.12538.

Several identification strategies

- Instrumental variable strategy
  - rely on the *exclusion restriction* (ER) assumption, which implies $\tau_{00} = \tau_{10}$ and NACE $= 0$
  - not appropriate if treatment receipt not strictly binary
    - compliance is defined using one important component of the intervention
    - dichotomizing a continuous participation variable
  - does not allow psychological effects, effects of compensating behaviors, etc.

- Principal ignorability (PI) strategy
  - PI assumption: $C \perp\!\!\!\perp Y_0 \mid X$, or $\text{E}[Y_0 \mid X, C = 1] = \text{E}[Y_0 \mid X, C = 0]$
  - appealing if have reach covariate data
  - justifies the *principal score weighting* method
  - does not assume NACE $= 0$, thus may be useful where ER is not justified

- Other strategies
  - Auxiliary independence: $W \perp\!\!\!\perp Y_0 \mid C, X$ for auxiliary variable W[2]
    - requires that such a special auxiliary variable exists

[2] Zhichao Jiang and Peng Ding. "Identification of Causal Effects Within Principal Strata Using Auxiliary Variables". In: *Statistical Science* 36.4 (2021), pp. 1–49. DOI: 10.1214/20-STS810.

Several identification strategies

- Instrumental variable strategy
  - rely on the *exclusion restriction* (ER) assumption, which implies $\tau_{00} = \tau_{10}$ and NACE $= 0$
  - not appropriate if treatment receipt not strictly binary
    - compliance is defined using one important component of the intervention
    - dichotomizing a continuous participation variable
  - does not allow psychological effects, effects of compensating behaviors, etc.

- Principal ignorability (PI) strategy
  - PI assumption: $C \perp\!\!\!\perp Y_0 \mid X$, or $\mathrm{E}[Y_0 \mid X, C = 1] = \mathrm{E}[Y_0 \mid X, C = 0]$
  - appealing if have reach covariate data
  - justifies the *principal score weighting* method
  - does not assume NACE $= 0$, thus may be useful where ER is not justified

- Other strategies
  - Auxiliary independence: $W \perp\!\!\!\perp Y_0 \mid C, X$ for auxiliary variable W
    - requires that such a special auxiliary variable exists
  - Principal strata sufficiently prognostic: $X \perp\!\!\!\perp Y_0 \mid C$
    - underlies methods that impute unobserved C based only on X or use the principal score in place of X in modeling Y_0
    - highly unrealistic

Our focus on PI

PI is an important identification assumption

But we need sensivity analyses – just like with any other untestable assumptions

Zooming in: the identification problem and the PI solution

$$\mathrm{E}[Y_z \mid C = c] = \frac{\mathrm{E}\{\overbrace{\mathrm{P}(C = c \mid X)}^{=:\,\pi_c(X)}\overbrace{\mathrm{E}[Y_z \mid X, C = c]}^{=:\,\mu_{zc}(X)}\}}{\mathrm{E}[\mathrm{P}(C = c \mid X)]}$$

Under A0, A1, A2

$\mathrm{E}[Y_1 \mid C = c]$ is identified because $\pi_c(X)$ and $\mu_{1c}(X)$ are identified

$$\pi_c(X) = \mathrm{P}(C = c \mid X, Z = 1)$$
$$\mu_{1c}(X) = \mathrm{E}[Y \mid X, Z = 1, C = c]$$

but $\mu_{0c}(X)$ is not identified so $\mathrm{E}[Y_0 \mid C = c]$ is not

Zooming in: the identification problem and the PI solution

The problem: one equation with two unknowns

$$\pi_1(X)\mu_{01}(X) + \pi_0(X)\mu_{00}(X) = \mu_0(X)$$

where $\mu_0(X) := \mathrm{E}[Y_0 \mid X]$, which under A0, A1, A2 is identified by $\mathrm{E}[Y \mid X, Z = 0]$

The solution given by the PI assumption:

$$\mu_{01}(X) = \mu_{00}(X) = \mu_0(X)$$

Existing sensitivity analyses for PI violation

Ding & Lu[2] use a mean ratio sens param $\frac{\mu_{01}(X)}{\mu_{00}(X)}$, and propose an estimator that under/over-weight the principal score – for the setting with randomized treatment

- when used with a binary (or bounded) outcome, this sens param may predict out of range
- this motivates us to develop methods that use a range of sens params

Wang et al.[3], when handling a survival outcome, uses a hazard ratio sens param in a Weibull (or Weibull mixture) model for $Y_0 \mid X, C$, and impute unobserved C and Y_0 as part of the fitting of a Bayesian joint model

[2] Peng Ding and Jiannan Lu. "Principal stratification analysis using principal scores". In: *Journal of the Royal Statistical Society. Series B: Statistical Methodology* 79.3 (2017), pp. 757–777. DOI: 10.1111/rssb.12191. arXiv: 1602.01196.

[3] Craig Wang et al. "Sensitivity analyses for the principal ignorability assumption using multiple imputation". en. In: *Pharmaceutical Statistics* 22.1 (2023), pp. 64–78. DOI: 10.1002/pst.2260. (Visited on 02/05/2023).

Outline

Organizing themes

2 sens analysis approaches

3 estimator types

Two sensitivity analysis approaches

anchoring on and deviating from

the mean and distribution versions of PI

$$\text{A3m:} \quad \mu_{01}(X) = \mu_{00}(X)$$

$$\text{A3d:} \quad C \perp\!\!\!\perp Y_0 \mid X$$

Three estimator types

A sensitivity analysis often follows and is secondary to a main analysis,
so consider sens analysis as modification of main analysis of different types

Three estimator types

A sensitivity analysis often follows and is secondary to a main analysis, so consider sens analysis as modification of main analysis of different types

- Type A estimators ($\approx$ outcome regression estimators)
  - estimates $\mu_0(X)$ to first estimate (non)complier effects conditional on covariates and then aggregates them to estimate CACE/NACE, eg

$$\frac{\sum_{i=1}^n \hat{\pi}_c(X_i)[\hat{\mu}_{1c}(X_i) - \hat{\mu}_0(X_i)]}{\sum_{i=1}^n \hat{\pi}_c(X_i)}, \quad \frac{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c)[Y_i - \hat{\mu}_0(X_i)]}{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c)}$$

  - sensitivity analysis technique: replace $\mu_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sensitivity assumption

Three estimator types

A sensitivity analysis often follows and is secondary to a main analysis,
so consider sens analysis as modification of main analysis of different types

- Type A estimators ($\approx$ outcome regression estimators)
  - estimates $\mu_0(X)$ to first estimate (non)complier effects conditional on covariates and then aggregates them to estimate CACE/NACE
  - sensitivity analysis technique: replace $\mu_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sensitivity assumption
- Type B estimators ($\approx$ influence function based estimators)
  - recall $\mathrm{E}[Y_1 \mid C = c] = \frac{\overbrace{\mathrm{E}[\pi_c(X)\mu_{1c}(X)]}^{\nu_{1c}}}{\underbrace{\mathrm{E}[\pi_c(X)]}_{\delta_c}}$, and $\mathrm{E}[Y_0 \mid C = c] \overset{\text{PI}}{=} \frac{\overbrace{\mathrm{E}[\pi_c(X)\mu_0(X)]}^{\nu_{0c}^{\text{PI}}}}{\underbrace{\mathrm{E}[\pi_c(X)]}_{\delta_c}}$
  - a type B estimator can be expressed as combination of IF-based estimators of δ_c, ν_{1c} and ν_{0c}^{PI}

$$\frac{\hat{\nu}_{1c,\text{IF}} - \hat{\nu}_{0c,\text{IF}}^{\text{PI}}}{\hat{\delta}_{c,\text{IF}}}$$

  - sensitivity analysis technique: replace $\hat{\nu}_{0c,\text{IF}}^{\text{PI}}$ with an IF-based estimator of ν_{0c} under the sensitivity assumption

Three estimator types

A sensitivity analysis often follows and is secondary to a main analysis, so consider sens analysis as modification of main analysis of different types

- Type A estimators ($\approx$ outcome regression estimators)
  - estimates $\mu_0(X)$ to first estimate (non)complier effects conditional on covariates and then aggregates them to estimate CACE/NACE
  - sensitivity analysis technique: replace $\mu_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sensitivity assumption
- Type B estimators ($\approx$ influence function based estimators)
  - can be expressed as combination of IF-based estimators of δ_c, ν_{1c} and ν_{0c}^{PI}
  - sensitivity analysis technique: replace $\hat{\nu}_{0c,\text{IF}}^{\text{PI}}$ with an IF-based estimator of ν_{0c} under the sensitivity assumption
- Type C estimators ($\approx$ other/weighting estimators)
  - do not estimate $\mu_0(X)$
  - an example is the pure weighting estimator

$$\frac{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c) Y_i}{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c)} - \frac{\sum_{i=1}^n \frac{1-Z_i}{\hat{e}(X_i,Z_i)} \hat{\pi}_c(X_i) Y_i}{\sum_{i=1}^n \frac{1-Z_i}{\hat{e}(X_i,Z_i)} \hat{\pi}_c(X_i)}$$

  - no specific sens analysis technique; consider case by case

Three estimator types

A sensitivity analysis often follows and is secondary to a main analysis, so consider sens analysis as modification of main analysis of different types

- Type A estimators ($\approx$ outcome regression estimators)
  - estimates $\mu_0(X)$ to first estimate (non)complier effects conditional on covariates and then aggregates them to estimate CACE/NACE
  - sensitivity analysis technique: replace $\mu_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sensitivity assumption
- Type B estimators ($\approx$ influence function based estimators)
  - can be expressed as combination of IF-based estimators of δ_c, ν_{1c} and ν_{0c}^{PI}
  - sensitivity analysis technique: replace $\hat{\nu}_{0c,\text{IF}}^{\text{PI}}$ with an IF-based estimator of ν_{0c} under the sensitivity assumption
- Type C estimators ($\approx$ other/weighting estimators)
  - do not estimate $\mu_0(X)$
  - no specific sens analysis technique; consider case by case

Outline

Three ratio-type sens params

A3m (PI, mean version): $\mu_{01}(X) = \mu_{00}(X)$.

A4-OR (sens odds ratio): $\dfrac{\mu_{01}(X)/[1-\mu_{01}(X)]}{\mu_{00}(X)/[1-\mu_{00}(X)]} = \rho$,

A4-GOR (sens generalized odds ratio): $\dfrac{[\mu_{01}(X)-l]/[h-\mu_{01}(X)]}{[\mu_{00}(X)-l]/[h-\mu_{00}(X)]} = \rho$

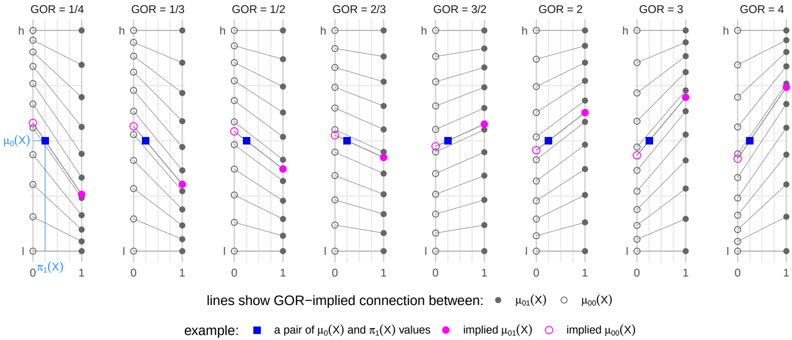
where l and h are the lower and upper bounds of Y_0,

A4-MR (sens mean ratio): $\dfrac{\mu_{01}(X)}{\mu_{00}(X)} = \rho$,

for some range of ρ that is considered plausible.

for binary outcomes (OR), outcomes bounded on both ends (GOR), and single-signed and otherwise unbounded outcomes (MR)

Connection between $\mu_{00}(X)$ and $\mu_{01}(X)$ under A4-GOR for different GOR values



Identification

Let $\rho_1 = \rho$, $\rho_0 = 1/\rho$

Under A4-OR combined with A0-A2,

$$\mu_{0c}(X) = \begin{cases} \frac{\overbrace{[\pi_c(X)+\mu_0(X)](\rho_c-1)+1}^{*} - \sqrt{*^2 - 4\pi_c(X)\mu_0(X)\rho_c(\rho_c-1)}}{2(\rho_c-1)\pi_c(X)} & \text{if } \rho_c \neq 1 \\ \mu_0(X) & \text{if } \rho_c = 1 \end{cases}$$

Under A4-GOR combined with A0-A2,

$$\mu_{0c}(X) = \begin{cases} \frac{\overbrace{[\pi_c(X)+\mu_0^\diamond(X)](\rho_c-1)+1}^{*} - \sqrt{*^2 - 4\pi_c(X)\mu_0^\diamond(X)\rho_c(\rho_c-1)}}{2(\rho_c-1)\pi_c(X)}(h-l)+l & \text{if } \rho_c \neq 1 \\ \mu_0(X) & \text{if } \rho_c = 1 \end{cases}$$

where $\mu_0^\diamond(X) := [\mu_0(X)-l]/(h-l)$

Under A4-MR combined with A0-A2,

$$\mu_{0c}(X) = \frac{\rho_c \mu_0(X)}{(\rho_c - 1)\pi_c(X) + 1}$$

Sensitivity analysis for the three estimator types

All three assumptions allow using the techniques of replacing $\mu_0(X)$ and ν_{0c}^{PI} for types A ($\approx$ outcome regression) and type B ($\approx$ IF-based) estimators

Only A4-MR provides for a simple modification of type C estimators: scale Y in control units by a factor of $\frac{\rho_c}{(\rho_c - 1)\pi_c(X) + 1}$

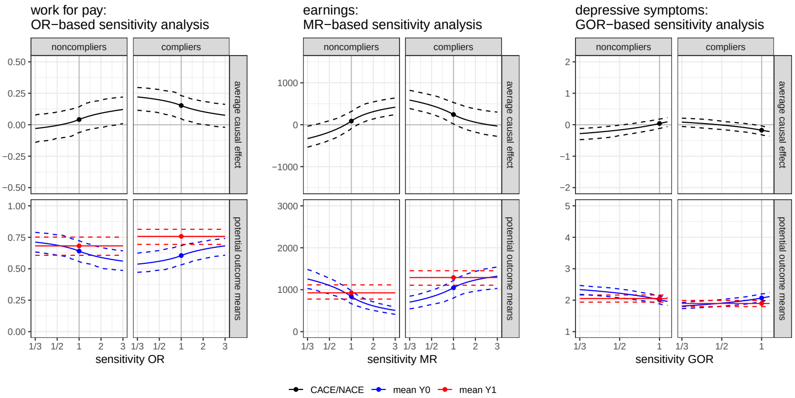
JOBS II illustration

PI-based analysis results: point estimates (95% BCa confidence intervals)

		compliers			noncompliers		
outcome	variable type	mean Y_1 (τ_{11})	mean Y_0 (τ_{01}^{PI})	CACE (Δ_1^{PI})	mean Y_1 (τ_{10})	mean Y_0 (τ_{00}^{PI})	NACE (Δ_0^{PI})
work	binary	75.4%	61.1%	14.3 percentage points	68.5%	64.2%	4.3 percentage points
		(69.4, 81.4)	(53.1, 68.4)	(4.7, 23.1)	(60.7, 75.2)	(55.7, 72.2)	(-6.2, 14.2)
earnings	actual range	$1,279	$1,014	$266	$928	$835	$92
	$0–5,667	(1,107, 1,452)	(802, 1,221)	(18, 530)	(776, 1,115)	(666, 972)	(-90, 318)
depressive	scale range	1.90	2.07	-0.18	2.05	2.02	0.02
symptoms	1 to 5	(1.80, 1.99)	(1.96, 2.20)	(-0.32, -0.04)	(1.94, 2.16)	(1.88, 2.14)	(-0.12, 0.18)

- for compliers, the intervention increased work and earnings, and decreased depressive symptoms
- for noncompliers, effects are close to null and non-sig

JOBS II illustration



caution about using the MR sens param (if time permits)

One difference-type sens param

Let $\sigma^2_{0c}(X) := \text{var}(Y_0 \mid X, C = c)$ and $\sigma^2_0(X) := \text{var}(Y_0 \mid X)$

A4-SMD (sens std mean difference): $$\frac{\mu_{01}(X) - \mu_{00}(X)}{\sqrt{(1/2)\sigma^2_{01}(X) + (1/2)\sigma^2_{00}(X)}} = \eta$$
for a range of η considered plausible

This assumption only partially identify $\mu_{0c}(X)$, so we consider supplementing it with an equal variance assumption

A4-SMDe (SMD, equal variance): A4-SMD, and $\sigma^2_{01}(X) = \sigma^2_{00}(X)$

Identification and estimation

Under A4-SMDe combined with A0-A2,

$$\mu_{0c}(X) = \mu_0(X) + (2c-1)\frac{\eta\pi_{1-c}(X)\sigma_0(X)}{\sqrt{1+\eta^2\pi_1(X)\pi_0(X)}},$$

$$\Delta_c = \Delta_c^{\text{PI}} - (2c-1)\eta\underbrace{\mathsf{E}\left[\frac{\pi_1(X)\pi_0(X)\sigma_0(X)}{\sqrt{1+\eta^2\pi_1(X)\pi_0(X)}}\right]}_{=:\,\lambda}/\delta_c =: \Delta_c^{\text{SMDe}}.$$

(If don't assume equal variance, get bounds instead of point identification)

In this case, sens analysis focuses on estimating $\Delta_c^{\text{PI}} - \Delta_c^{\text{SMDe}}$

use plug-in or IF-based estimation

Outline

The sens assumptions so far are about the conditional means $\mu_{0c}(X) := \mathsf{E}[Y_0 \mid X, C = c]$

For continuous outcomes, there may be values of the sens parameter that conflict with the distribution of $Y_0 \mid X$

Now we take an approach that is fully informed by this distribution

Recall the distribution verion of PI

A3d: $C \perp\!\!\!\perp Y_0 \mid X$

Want an assumption that allows Y_0 and C to be dependent given X
that helps identify $\mu_{0c}(X)$

The usual exponential tilting technique if applied would relate the probability of C (given X, Y_0) to a simple function of Y_0 and the sens param

but that would not be enough to identify the components of the mixture

2-step construction

Let $\tilde{\pi}_1(X, Y_0) := \mathrm{P}(C = 1 \mid X, Y_0)$. Use shorthand $\tilde{\pi}_1$

Step 1: Assume a distribution for $\tilde{\pi}_1$ given X with mean $\pi_1(X)$ that allows $\tilde{\pi}_1$ to vary, indexed by a dispersion param

Step 2: Connect Y_0 to $\tilde{\pi}_1$ within the confines of that distribution to induce Y_0-C association

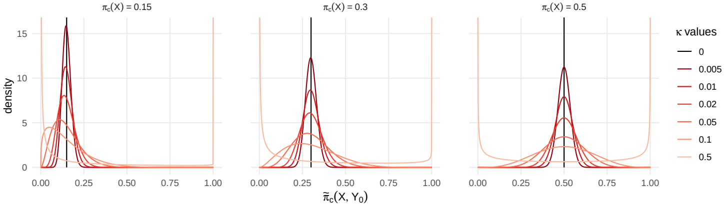
2-step construction

Let $\tilde{\pi}_1(X, Y_0) := \mathrm{P}(C = 1 \mid X, Y_0)$. Use shorthand $\tilde{\pi}_1$

Step 1: Assume a distribution for $\tilde{\pi}_1$ given X with mean $\pi_1(X)$ that allows $\tilde{\pi}_1$ to vary, indexed by a dispersion param

- borrow Victor Veitch's idea of using the beta distribution[4]

$$\text{Beta}\left(\pi_1(X)\frac{1-\kappa}{\kappa}, \pi_0(X)\frac{1-\kappa}{\kappa}\right)$$



[4] Victor Veitch and Anisha Zaveri. "Sense and Sensitivity Analysis: Simple Post-Hoc Analysis of Bias Due to Unobserved Confounding". In: (2020). arxiv: 2003.01747. arXiv: 2003.01747.

2-step construction

Let $\tilde{\pi}_1(X, Y_0) := \mathrm{P}(C = 1 \mid X, Y_0)$. Use shorthand $\tilde{\pi}_1$

Step 1: Assume a distribution for $\tilde{\pi}_1$ given X with mean $\pi_1(X)$ that allows $\tilde{\pi}_1$ to vary, indexed by a dispersion param

- borrow Victor Veitch's idea of using the beta distribution

$$\text{Beta}\left(\pi_1(X)\frac{1-\kappa}{\kappa}, \pi_0(X)\frac{1-\kappa}{\kappa}\right)$$

Step 2: Connect Y_0 to $\tilde{\pi}_1$ within the confines of that distribution to induce Y_0-C association

- use quantile-to-quantile mapping
  - either same-quantiles or opposite-quantiles

2-step construction

Let $\tilde{\pi}_1(X, Y_0) := \text{P}(C = 1 \mid X, Y_0)$. Use shorthand $\tilde{\pi}_1$

Step 1: Assume a distribution for $\tilde{\pi}_1$ given X with mean $\pi_1(X)$ that allows $\tilde{\pi}_1$ to vary, indexed by a dispersion param

- borrow Victor Veitch's idea of using the beta distribution

$$\text{Beta}\left(\pi_1(X)\frac{1-\kappa}{\kappa}, \pi_0(X)\frac{1-\kappa}{\kappa}\right)$$

Step 2: Connect Y_0 to $\tilde{\pi}_1$ within the confines of that distribution to induce Y_0-C association

- use quantile-to-quantile mapping
  - either same-quantiles or opposite-quantiles

A5 (sens beta quantile):

$$B_{X,\kappa}(\tilde{\pi}_1) = \begin{cases} F_{Y_0|X}(Y_0) & \text{if assume positive } Y_0\text{-}C \text{ association} \\ 1 - F_{Y_0|X}(Y_0) & \text{if assume negative } Y_0\text{-}C \text{ association} \end{cases}$$

Identification and estimation

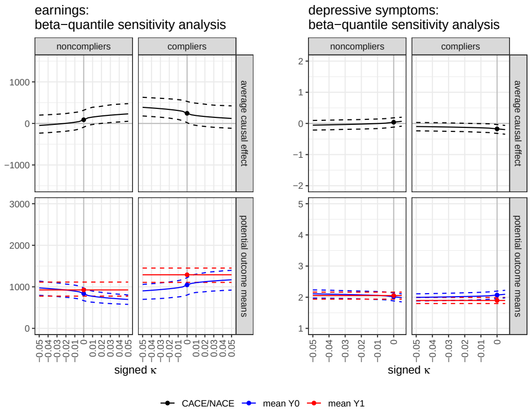
Under A5 combined with A0-A2,

$$\mu_{0c}(X) = \frac{\mathrm{E}[\tilde{\pi}_c Y \mid X, Z = 0]}{\mathrm{E}[\tilde{\pi}_c \mid X, Z = 0]},$$

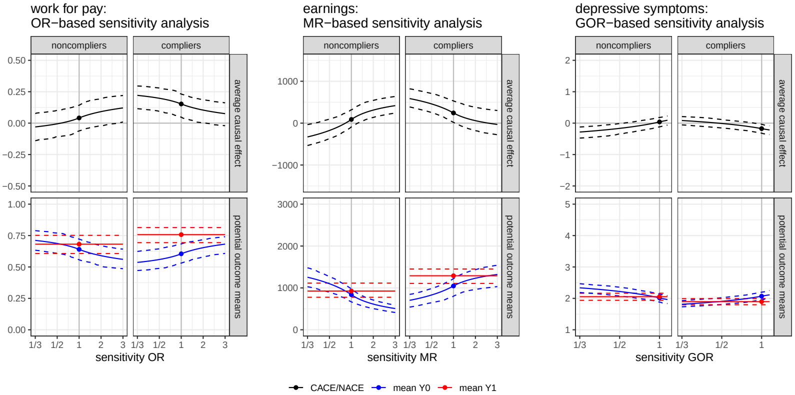
where $\tilde{\pi}_c$ and Y are quantile-to-quantile connected

We estimate $\mathrm{P}(Y \mid X, Z = 0)$ and then estimate $\mu_{0c}(X)$ using numerical integration

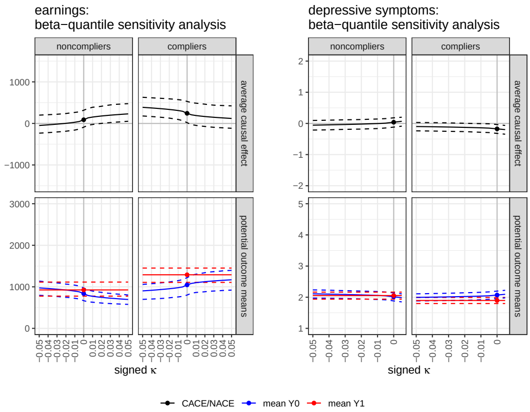
JOBS II illustration



JOBS II illustration



JOBS II illustration



Results are less extreme than mean-based sens analysis

Outline

We have expanded solutions for sens analysis for PI violation in estimating complier and noncomplier average causal effects

THANK YOU!