Sensitivity analysis for principal ignorability violation in estimating complier and noncomplier average causal effects

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joint work with Liz Stuart, Dan Scharfstein, Betsy Ogburn
(all problems mine)

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Noncompliance and principal stratification

The study of treatment effects often complicated by noncompliance
or other significant post-treatment events

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or effect of treatment given a post-treatment event

But would break randomization

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Principal stratification (Frangakis & Rubin 2002)
avoids this problem by creating a new pre-treatment variable
and considering effects stratified on it

Notation

Z	treatment assigned (binary)
Y	outcome
Y_1, Y_0	potential outcomes (potential values of Y)
X	baseline covariates
S	treatment received or post-treatment event (here binary)
C	principal stratum, defined based on potential values (S_1, S_0) of S

Principal causal effects: $\mathrm{E}[Y_1 - Y_0 \mid C = c]$, where c is a value of C

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- Y well-defined generally
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Example: JOBS II

- unemployed workers are randomized to *intervention* (training to improve job searching skills and mental health) or *control* (booklet with job search tips)
- about half of those assigned to intervention didn't attend
- outcomes: work, earnings, depressive symptoms

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Two principal strata (aka compliance types)

- those who *would* attend if offered the intervention, aka *compliers* ($C = 1$)
- those who *would not* attend if offered the intervention, aka *noncompliers* ($C = 0$)

Estimands: complier and noncomplier average causal effects (CACE, NACE)

$$\Delta_c := \mathrm{E}[Y_1 - Y_0 \mid C = c] \text{ for } c = 1, 0$$

The identification challenge: C is partially observed

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- IV strategy (ER assumption): treatment assigned affects outcome only through treatment received
  - ie no effects on noncompliers

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Several identification strategies

- IV strategy (ER assumption): treatment assigned affects outcome only through treatment received
  - ie no effects on noncompliers
- Principal ignorabilty (PI): conditional on X, principal stratum does not carry info about outcome under control
  - appealing if don't want to assume NACE=0 and rich covariates
  - $\mathrm{E}[Y \mid X, Z = 0, C = 0] = \mathrm{E}[Y \mid X, Z = 0, C = 1]$
- etc.

Our focus on PI

$$\mathrm{E}[Y \mid X, Z = 0, C = 1] = \mathrm{E}[Y \mid X, Z = 0, C = 0]$$

Need sensitivity analyses – just like with any other untestable assumptions

Inspiring prior work

Ding and Lu (2017) use a mean ratio sensitivity parameter

$$\frac{\mathrm{E}[Y \mid X, Z = 0, C = 1]}{\mathrm{E}[Y \mid X, Z = 0, C = 0]} = \rho$$

and modify a PI-based weighting estimator to incorporate ρ

Objective

Develop sensitivity analyses for PI violation that

- use a range of sens params
- handle a range of estimators

Some shorthand notation

Stratum-specific outcome means

$$\mu_{zc}(X) := \mathrm{E}[Y \mid X, Z = z, C = c]$$

in this notation, PI is

$$\mu_{01}(X) = \mu_{00}(X)$$

Some shorthand notation

Stratum-specific outcome means and variances

$$\mu_{zc}(X) := \mathrm{E}[Y \mid X, Z = z, C = c]$$

$$\sigma^2_{zc}(X) := \mathrm{var}(Y \mid X, Z = z, C = c)$$

Stratum-agnostic outcome means and variances

$$\kappa_0(X) := \mathrm{E}[Y \mid X, Z = 0]$$

$$\zeta^2_0(X) := \mathrm{var}(Y \mid X, Z = 0)$$

Principal scores

$$\pi_c(X) := \mathrm{P}(C = c \mid X, Z = 1)$$

Propensity scores

$$e(X, Z) = \mathrm{P}(Z \mid X)$$

A range of sens assumptions with different sens params

Recall PI: $\mu_{01}(X) = \mu_{00}(X)$.

Sens assumptions:

sens-MR: $$\frac{\mu_{01}(X)}{\mu_{00}(X)} = \rho,$$

sens-OR: $$\frac{\mu_{01}(X)/[1-\mu_{01}(X)]}{\mu_{00}(X)/[1-\mu_{00}(X)]} = \psi,$$

sens-GOR: $$\frac{[\mu_{01}(X)-l]/[h-\mu_{01}(X)]}{[\mu_{00}(X)-l]/[h-\mu_{00}(X)]} = \psi$$

where l and h are the lower and upper outcome bounds,

sens-SMD: $$\frac{\mu_{01}(X)-\mu_{00}(X)}{\sqrt{[\sigma_{01}^2(X)+\sigma_{00}^2(X)]/2}} = \eta,$$

for some range of ρ, ψ or η that is considered plausible.

PI-based identification of Δ_c

Under unconfoundedness

$$E[Y_z \mid C = c] = \frac{E[\pi_c(X)\mu_{zc}(X)]}{E[\pi_c(X)]}$$

where $\mu_{1c}(X)$ is an observed data function,
but $\mu_{0c}(X)$ is an unknown in the mixture equation

$$\pi_1(X)\mu_{01}(X) + \pi_0(X)\mu_{00}(X) = \kappa_0(X).$$

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Hence under PI + unconfoundedness

$$\Delta_c = \frac{\mathrm{E}\{\pi_c(X)[\mu_{1c}(X) - \kappa_0(X)]\}}{\mathrm{E}[\pi_c(X)]}.$$

For sens analysis, need the sens assumption to help solve the mixture equation.

PI-based estimation: 3 estimator types from a sens analysis perspective

Sens analysis = a modification of main analysis

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Sens analysis = a modification of main analysis

- Type A ($\approx$ outcome regression estimators)
  - estimates $\kappa_0(X)$ to first estimate effects conditional on covariates and then aggregates them to estimate CACE/NACE, eg

$$\frac{\sum_{i=1}^n \hat{\pi}_c(X_i)[\hat{\mu}_{1c}(X_i) - \hat{\kappa}_0(X_i)]}{\sum_{i=1}^n \hat{\pi}_c(X_i)}, \quad \frac{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c)[Y_i - \hat{\kappa}_0(X_i)]}{\sum_{i=1}^n \frac{Z_i}{\hat{e}(X_i,Z_i)} \mathrm{I}(C_i = c)}$$

  - sens analysis technique: replace $\kappa_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sens assumption

PI-based estimation: 3 estimator types from a sens analysis perspective

Sens analysis = a modification of main analysis

- Type B ($\approx$ influence function based estimators)
  - write
    $$\Delta_c^{\mathsf{PI}} = \frac{\nu_{1c} - \nu_{0c}^{\mathsf{PI}}}{\pi_c}$$
    where $\nu_{zc} := \mathsf{E}[\pi_c(X)\mu_{zc}(X)]$, $\nu_{0c}^{\mathsf{PI}} := \mathsf{E}[\pi_c(X)\kappa_0(X)]$, $\pi_c := \mathsf{E}[\pi_c(X)]$
  - a type B estimator can be expressed as combination of IF-based estimators of π_c, ν_{1c} and ν_{0c}^{PI}
    $$\frac{\hat{\nu}_{1c,\text{IF}} - \hat{\nu}_{0c,\text{IF}}^{\mathsf{PI}}}{\hat{\delta}_{c,\text{IF}}}$$
  - sens analysis technique: replace $\hat{\nu}_{0c,\text{IF}}^{\mathsf{PI}}$ with an IF-based estimator of ν_{0c} under the sens assumption

PI-based estimation: 3 estimator types from a sens analysis perspective

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- Type C (≈ other/weighting estimators)
  - an example is the pure weighting estimator

$$\frac{\sum_{i=1}^{n} \frac{Z_i}{\hat{e}(X_i, Z_i)} \mathrm{I}(C_i = c) Y_i}{\sum_{i=1}^{n} \frac{Z_i}{\hat{e}(X_i, Z_i)} \mathrm{I}(C_i = c)} - \frac{\sum_{i=1}^{n} \frac{1 - Z_i}{\hat{e}(X_i, Z_i)} \hat{\pi}_c(X_i) Y_i}{\sum_{i=1}^{n} \frac{1 - Z_i}{\hat{e}(X_i, Z_i)} \hat{\pi}_c(X_i)}$$

  - no specific sens analysis technique; consider case by case

PI-based estimation: 3 estimator types from a sens analysis perspective

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- Type A (≈ outcome regression estimators)
  - estimates $\kappa_0(X)$ to first estimate effects conditional on covariates and then aggregates them to estimate CACE/NACE
  - sens analysis technique: replace $\kappa_0(X)$ by the identification result of $\mu_{0c}(X)$ under the sens assumption
- Type B (≈ influence function based estimators)
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- Type C (≈ other/weighting estimators)
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Under sens-GOR and sens-OR

Let $\psi_1 = \psi$, $\psi_0 = 1/\psi$

Under sens-GOR,

$$\mu_{0c}(X) = \begin{cases} \dfrac{\overbrace{[\pi_c(X)+\kappa_0^\diamond(X)](\psi_c-1)+1}^{*} - \sqrt{*^2 - 4\pi_c(X)\kappa_0^\diamond(X)\psi_c(\psi_c-1)}}{2(\psi_c-1)\pi_c(X)}(h-l)+l & \text{if } \psi_c \neq 1 \\ \kappa_0(X) & \text{if } \psi_c = 1 \end{cases}$$

where $\kappa_0^\diamond(X) := [\kappa_0(X) - l]/(h-l)$

Under sens-OR,

$$\mu_{0c}(X) = \begin{cases} \dfrac{\overbrace{[\pi_c(X)+\kappa_0(X)](\psi_c-1)+1}^{*} - \sqrt{*^2 - 4\pi_c(X)\kappa_0(X)\psi_c(\psi_c-1)}}{2(\psi_c-1)\pi_c(X)} & \text{if } \psi_c \neq 1 \\ \kappa_0(X) & \text{if } \psi_c = 1 \end{cases}$$

These results allow using the techniques of replacing $\kappa_0(X)$ and ν_{0c}^{PI} for type A and type B estimators

Under sens-MR

Let $\rho_1 = \rho,\ \rho_0 = 1/\rho$

Under sens-MR,

$$\mu_{0c}(X) = \frac{\rho_c}{(\rho_c - 1)\pi_c(X) + 1}\kappa_0(X)$$

This result allows modifying estimators of all three types

(for type C, scale the outcome in those under control)

Under sens-SMD and variants

Let $\eta_1 = \eta, \eta_0 = -\eta$

Under sens-SMD, Δ_c lies between the two bounds:

$$\Delta_c^{\mathsf{PI}} - \eta_c \, \mathsf{E}\left[\frac{\pi_1(X)\pi_0(X)\zeta_0(X)}{\sqrt{1 \pm |\pi_1(X) - \pi_0(X)| + \eta^2 \pi_1(X)\pi_0(X)}} \right] / \pi_c .$$

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If supplement with the assumption that $\frac{1}{k} \leq \frac{\sigma_{01}^2(X)}{\sigma_{00}^2(X)} \leq k$ for a specified $k > 1$ (sens-SMDr), the bounds are tightened:

$$\Delta_c^{\mathrm{PI}} - \eta_c \, \mathrm{E}\left[\frac{\pi_1(X)\pi_0(X)\zeta_0(X)}{\sqrt{1 \pm \frac{k-1}{k+1}|\pi_1(X) - \pi_0(X)| + \eta^2 \pi_1(X)\pi_0(X)}}\right] / \pi_c.$$

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If supplement with the assumption that $\sigma_{01}^2(X) = \sigma_{00}^2(X)$ (sens-SMDe), achieve point identification:

$$\Delta_c^{\mathrm{PI}} - \eta_c \mathrm{E}\left[\frac{\pi_1(X)\pi_0(X)\zeta_0(X)}{\sqrt{1 + \eta^2\pi_1(X)\pi_0(X)}}\right] / \pi_c.$$

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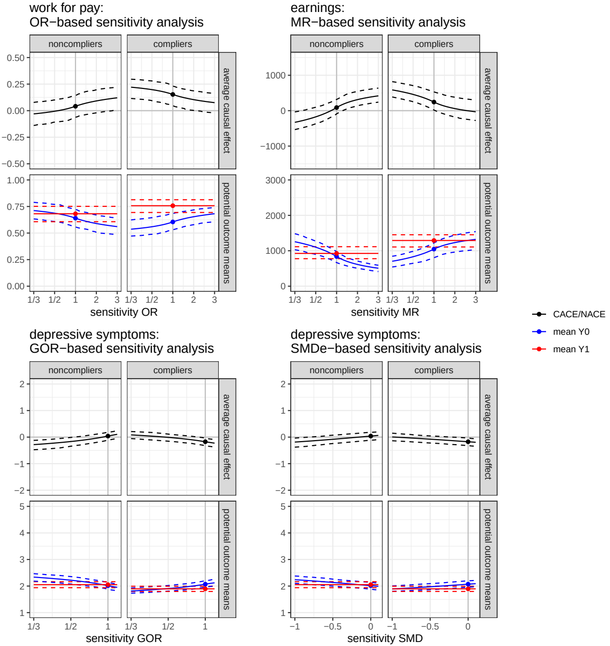
Sens analysis focuses on estimating the difference term

JOBS II results



Some things we noticed

- Partial loss of multiple robustness

  because $\mu_{0c}(X)$ is a function of $\pi_c(X)$ and $\kappa_0(X)$

- A pattern of finite-sample bias for the sens analysis where effect estimates are less extreme than should be

  because $\mathrm{E}[Y_0 \mid C = c] = \dfrac{\mathrm{E}[\pi_c(X)\mu_{0c}(X)]}{\mathrm{E}[\pi_c(X)]}$ is a weighted average where the function being averaged depends on the weight

To sum up

Expanded options for sens analysis for PI violation in CACE/NACE estimation

Next step

Risk of contradicting the observed data distribution

- sens-OR (binary Y): none
- sens-MR: substantial
- sens-GOR and sens-SMD: reduced

On-going work: a sens analysis fully informed by $\mathrm{P}(Y \mid X, Z = 0)$

THANK YOU!