Propensity score weighting analysis with complex survey data: when treatment effects are heterogeneous across strata and clusters

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Outline

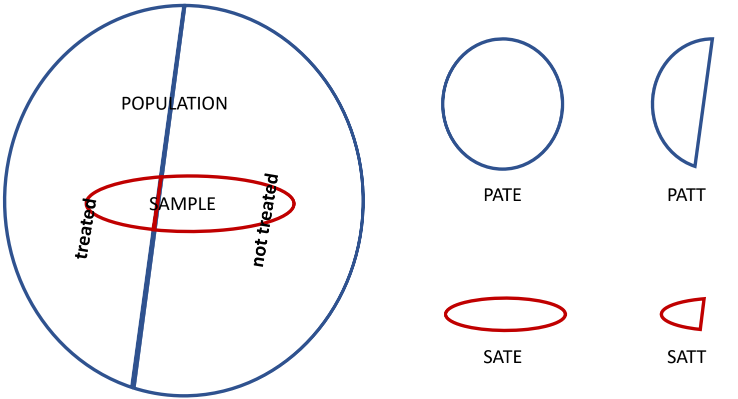
Outline

Big picture

- Researchers may be interested in making causal statements about populations – relevant for policy recommendations
    - What "works" in general practice?
    - What "works" for the general population?
- Ideal: a randomized trial in a representative sample. Rare!
- Instead we have the trade-off:
    - Randomized trials: unbiased for sample, but selective populations
    - Non-experimental studies: data on broad populations, but selection bias

Population vs. sample effects

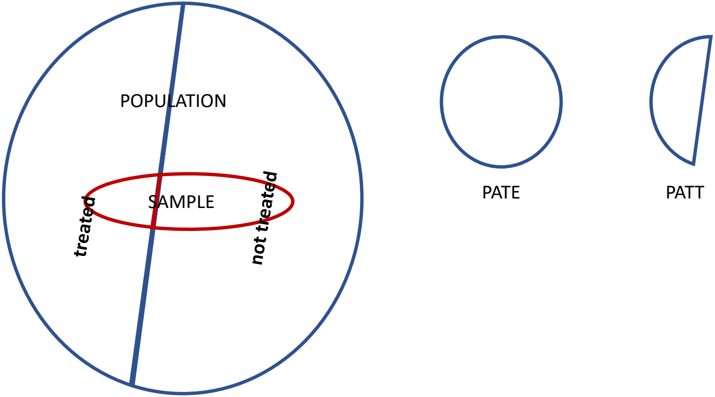
ATE = average treatment effect; ATT = average treatment effect on the treated



Estimating population effects

How to use a representative yet complex sample to estimate population effects?

– eg the Early Childhood Longitudinal Studies, the Add Health Study



Outline

Propensity scores (PS)

- To infer effect of treatment A on outcome Y: need treated ($A = 1$) and comparison ($A = 0$) groups to be comparable
  - Not in observational studies
  - So, make them look similar on observed characteristics X – those that may confound treatment effects
  - Assume no unmeasured confounders U, i.e., potential outcomes $(Y^{(1)}, Y^{(0)}) \perp\!\!\!\perp A|X$
- PS = probability of receiving treatment, given covariates X (Rosenbaum & Rubin, 1983)
  - Is "balancing score", ie given PS, distribution of X is the same between treated and comparison
  - Use the estimated PS to balance covariate distribution: matching, weighting, subclassification
- After balance obtained
  - Compare outcome between balanced treated and comparison groups
  - Or fit an outcome model (w/ covariates) to the balanced sample

PS methods and complex samples

- Using PS methods on representative population datasets should get us population treatment effects
- But original PS methods assume simple random sampling
  - Many applications with complex survey data ignore survey weights (DuGoff et al., 2014)
- PS methods for complex samples still open area of research
  - how to incorporate survey weights
  - how to handle strata and clusters

PSs and complex samples: survey weights

- Zanutto (2006), Dugoff et al. (2014), Ridgeway et al. (2015), Austin et al. (2016), Lenis et al. (2017)

- Assuming no U, my read from this literature:

  - Use survey weights for PS model? It depends.
    - PS matching: no need to incorporate survey weights
    - PS weighting: generally, survey-weight the PS model (more in a bit!)

  - Use survey weights for outcome model? Yes!
    - PS matching: survey-weight the outcome model + transfer survey weight when $S \not\perp\!\!\!\perp A|X$
    - PS weighting: multiply survey weights and PS weights

PSs and complex samples: strata and clusters

- Strata, clusters as design features: include in survey analysis commands (eg when fitting outcome model) for variance estimation

- Strata as analysis variable: include stratum indicators as predictors in outcome model

- Clusters: there is a relevant literature on multilevel PS methods, motivated by clustered data (not necessarily complex surveys)
  – see Hong & Raudenbush 2006, Arpino & Mealli 2011, Kelcey 2011, Thoemmes & West 2011, Li et al. 2013
  - Treatment assignment model may be multilevel with influences by covariates at cluster/individual levels and random effects
  - Outcome model may be multilevel as well

Outline

Motivation

- Interested in using PS weighting to estimate PATE
  - simple translation for PATT
- Multi-stage sample: several strata Z, with clusters C nested in strata, and individuals nested in clusters
- Concerned about heterogeneity associated with strata and clusters

Strata-related heterogeneity

- Two of the reasons for using stratified sampling instead of SRS:
  - to ensure enough representation of each stratum (subpopulation)
  - to reduce variance of estimates, because within-stratum variance is believed to be smaller than total variance
- Both imply potentially important/substantial differences across strata

- Our concern: strata may be systematically different with respect to
  - covariate distribution
  - treatment assignment: prevalence of A, influence of X on A
  - treatment effects: average effects, X's modification of effects
- An otherwise appropriate PS analysis that simply treats Z as a design feature in fitting models might be biased

Clusters-related heterogeneity

- Clusters within a stratum may also vary in the same aspects
- Assume such variation within a stratum is random
  - same spirit with the assumption that sampling units are exchangeable

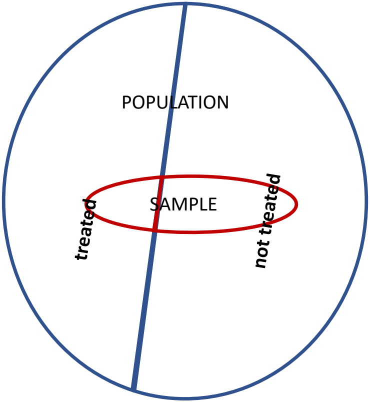
Treatment assignment and treatment effects

- Treatment assignment
  - True model $\text{P}(A = 1 \mid X, Z, C)$
  - Assume positivity, ie $0 < \text{P}(A = 1 \mid X, Z, C) < 1$
- Potential outcomes and treatment effects
  - True models $\text{P}[Y^{(a)} \mid X, Z, C]$, $a = 1, 0$
  - Assume no unmeasured confounders $(Y^{(1)}, Y^{(0)}) \perp\!\!\!\perp A \mid (X, Z, C)$ plus no interference, consistency
  - Estimand: $\text{PATE} = \text{E}[Y^{(1)} - Y^{(0)}]$

Sample participation and survey weights

- Multi-stage probability sampling
  - Clusters sampled within strata – probabilities depending on stratum and cluster
  - Units sampled within sampled clusters – usually with equal probability
- Non-response
  - May depend on factors/characteristics W at cluster or unit level
  - Surveys often adjust for non-response
- Sample participation S requires being sampled and responding
  - True model $\mathrm{P}(S = 1 \mid Z, C, W)$
  - Survey weights are $\dfrac{1}{\hat{\mathrm{P}}(S = 1 \mid Z = Z_i, C = C_i, W = W_i)}$

Weights for estimating population effects



To estimate PATE, need to weight sample treated and sample comparison groups to the population w.r.t. variables that influence $Y_i^{(a)}$ (or TE_i)

Weights for estimating population effects

- The weights that do this are the inverse of

$$\mathrm{P}(S = 1, A = A_i \mid X = X_i, Z = Z_i, C = C_i)$$

  - Case 1: if sampling happened after treatment assigned, factor

$$= \mathrm{P}(S = 1 \mid A = A_i, X_i, Z_i, C_i)\mathrm{P}(A = A_i \mid X_i, Z_i, C_i)$$

  - Case 2: if treatment assigned after sample assembled, factor

$$= \mathrm{P}(S = 1 \mid X_i, Z_i, C_i)\mathrm{P}(A = A_i \mid S = 1, X_i, Z_i, C_i)$$

- First piece: taken care of by survey weights, if $(A, X) \subset W$ or $X \subset W$
- Second piece: population PS in case 1, sample PS in case 2
  - case 1 requires survey-weighting the PS model, but not case 2

PSs need to be estimated

- If sample size of each cluster is large, can estimate within each cluster
- If not, need to use some model
- Consider Z first (assuming number of strata not large):
  - ignore strata – not very good
  - stratum indicators – better
  - stratified by stratum – probably best
- Consider C (assuming a lot of clusters):
  - use multilevel modeling – probably best
  - ignore clusters – maybe ok when the outcome model is linear, but not good otherwise
- Assume X is fully captured

Outline

Simulations to date

- For each scenario, generate 100 populations

- For each population, draw 10,000 samples

Population structure

stratum	number of clusters	cluster size
1	90	6000
2	60	6000
3	70	4000
4	80	4000
5	200	2000
6	150	2000

Covariate distribution

2 covariates at individual level

- binary X_1: prevalence varies
  - systematically across strata: .55, .35, .3, .7, .4, .6
  - randomly across clusters by a beta model

- continuous X_2:

$$X_{2i} = X_{1i} + U_c^{X_2} + \epsilon_i^{x_2}, \quad U_c^{X_2} \sim N(0, .2), \quad \epsilon_i^{x_2} \sim N(0, 1)$$

Treatment assignment

$$
\begin{aligned}
\text{logit}[\text{P}(A=1|X,Z,C)] = {} & [-.5 + (.3)\mathbb{1}\{Z=1,2\} - (.2)\mathbb{1}\{Z=5,6\} + U_c^{A1}] + \\
& [1 + (.5)\mathbb{1}\{Z=1,2\} - (.5)\mathbb{1}\{Z=5,6\} + U_c^{AX_1}]X_1 + \\
& [.5 + (.2)\mathbb{1}\{Z=1,2\} - (.2)\mathbb{1}\{Z=5,6\} + U_c^{AX_2}]X_2 +
\end{aligned}
$$

- Scenarios vary in the inclusion or exclusion of
  - strata main and interaction effects
  - random cluster effects (normal or recentered gamma)

Potential outcomes and treatment effects

$$
\begin{aligned}
Y(0) = {} & U_c^{Y_0} + \\
& X_1 + \\
& X_2 + \\
& \epsilon^{Y_0} \\
Y(1) = {} & U_c^{Y_0} + [(2)\mathbb{1}\{Z = 1, 2\} - (2)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}}] + \\
& X_1 + [1 + (.5)\mathbb{1}\{Z = 1, 2\} - (.5)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}X_1}]X_1 + \\
& X_2 + [1 + (.5)\mathbb{1}\{Z = 1, 2\} - (.5)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}X_2}]X_2 + \\
& \epsilon^{Y_1} \\
\text{TE} = {} & [(2)\mathbb{1}\{Z = 1, 2\} - (2)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}}] + \\
& [1 + (.5)\mathbb{1}\{Z = 1, 2\} - (.5)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}X_1}]X_1 + \\
& [1 + (.5)\mathbb{1}\{Z = 1, 2\} - (.5)\mathbb{1}\{Z = 5, 6\} + U_c^{\text{TE}X_2}]X_2 + \\
& \epsilon^{Y_1} - \epsilon^{Y_0}
\end{aligned}
$$

$\epsilon^{Y_1}, \epsilon^{Y_0} \sim N(0, 1)$. Random cluster effects are normal or recentered gamma.

Sample participation

- In all scenarios, S depends on Z and C via sampling design
  - base scenario: sample 10 clusters per stratum, 100 units per cluster

- Variation due to non-response
  - S does not depend on X or A (base scenario)
  - S depends on binary X_1
  - S depends on A

- Such dependence is captured in survey weights

Methods implemented

- So far, use one-level models, ignoring clusters
- 3 methods w.r.t. strata
  - Naive: ignore strata in both PS and outcome models
  - Strata as covariates: include stratum indicators in PS and outcome models
  - Stratified analysis: fit PS model, balance covariates, and fit outcome model in each stratum separately and then combine
- All models fit using `survey` package, with strata, clusters and weights as design features

Results

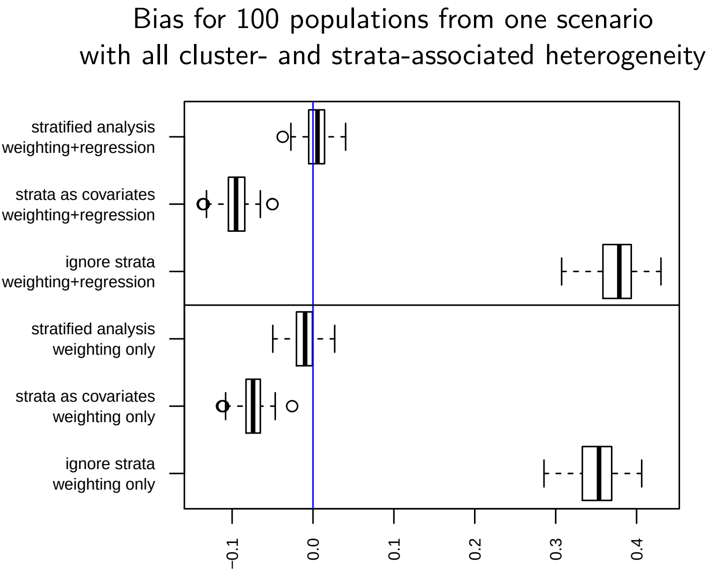
- Variation in model for sample participation does not matter
  - Not surprising as we have correct survey weights
- Random cluster effects of all kinds only increase variance and do not affect bias
  - Because our outcome model is linear – biases in weights lead to biases contributed by individuals to the PATE that average to zero
  - May not be the case with a nonlinear outcome model
  - Then might want to use a multilevel model to better estimate the PSs
  - Also, a multilevel outcome model may help reduce variance

Results

- When treatment effects vary across strata, the naive method is biased
  - Because naive method does not balance Z

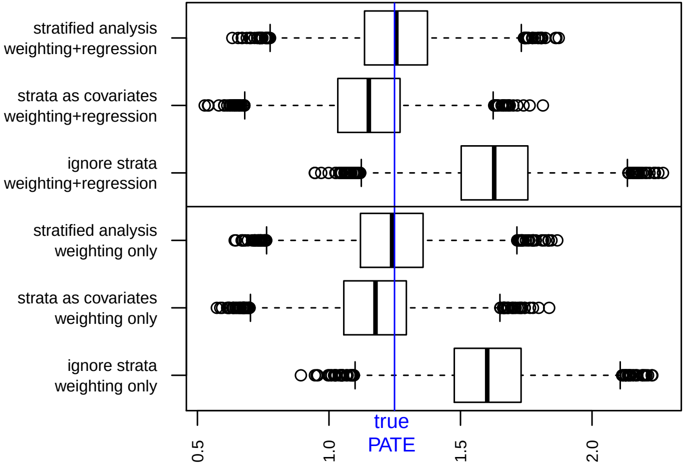
- When covariates' influence on treatment assignment also varies across strata, the strata-as-covariates method is also biased, but stratified analysis remains unbiased

Results


Bias for 100 populations from one scenario
with all cluster- and strata-associated heterogeneity
stratified analysis
weighting+regression
strata as covariates
weighting+regression
ignore strata
weighting+regression
stratified analysis
weighting only
strata as covariates
weighting only
ignore strata
weighting only
−0.1
0.0
0.1
0.2
0.3
0.4

Results

Estimates on 10,000 samples
drawn from one of those populations



Outline

Recommendations: how to handle strata

- When strata are suspected to vary with respect to either treatment effect or treatment assignment model, they should be incorporated in the analysis
- If strata are suspected to interact with covariates in influencing treatment assignment, stratified analysis is preferred

Recommendations: weights when using PS weighting

- Multiply weights: survey weight × PS weight
- Decide whether PS weight should be based on population PS or sample PS – depends on what the survey weight captures

$$\text{PATE-weight}_i = [\mathrm{P}(S=1, A=A_i \mid X=X_i, Z=Z_i, C=C_i)]^{-1}$$

$$= \begin{cases} \underbrace{[\mathrm{P}(S=1 \mid A_i, X_i, Z_i, C_i)]^{-1}}_{\text{does survey weight capture this?}} \times \underbrace{[\mathrm{P}(A=A_i \mid X_i, Z_i, C_i)]^{-1}}_{\text{population PS}} & \text{case 1} \\ \underbrace{[\mathrm{P}(S=1 \mid X_i, Z_i, C_i)]^{-1}}_{\text{or does it capture this?}} \times \underbrace{[\mathrm{P}(A=A_i \mid S=1, X_i, Z_i, C_i)]^{-1}}_{\text{sample PS}} & \text{case 2} \end{cases}$$

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Thank you!